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COMPLEX SUBSPACES OF HOMOGENEOUS COMPLEX MANIFOLDS
I. TRANSPLANTING THEOREMS

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To the memory of my father

In [3] Barth proved a number of results showing that complex submanifolds of $\mathbf{P}^N_{\mathbf{C}}$ of small codimension resemble $\mathbf{P}^N_{\mathbf{C}}$ cohomologically. These results, which go under the general title of transplanting theorems, are generalized in this paper to arbitrary homogeneous complex manifolds.

The research announcement [17] serves as a general introduction to the family [18, 19, 20] of papers to which this paper belongs.

In §0 I give some notation and recall some basic results of Andreotti-Grauert [1] on convexity.

In §1 I prove a parametrized version (1.1–1.3) of these results. This local transplanting theorem gives conditions when a sheaf cohomology class defined in a neighborhood of an analytic subspace Y of an analytic space X, can be 'transplanted' to neighborhoods of analytic subspaces of X obtained from Y by moving in a 'continuous family' of analytic subspaces, Y_t, of X.

In §2 I prove some spectral sequence lemmas. These are used in §3 to study when the above transplants harmonize into a global cohomology class.

In §3 are the main theorems of this paper. The following corollary gives the flavour of my results.

COROLLARY. *Let A and B be complex submanifolds of a simple Abelian variety, X, i.e., X is an Abelian variety without proper sub-Abelian varieties. Then*:

(a) $H^j(A, A \cap B, \mathbf{C}) = 0$ *for* $j \leqslant \min\{\dim_{\mathbf{C}} B + 1, \dim_{\mathbf{C}} A\} - \operatorname{cod}_{\mathbf{C}} B$, *and,*

(b) *given any coherent analytic sheaf* $\mathcal{S}$ *on* A, $H^j(A - A \cap B, \mathcal{S}) = 0$ *for* $j \geqslant \operatorname{cod}_{\mathbf{C}} B + \max\{0, \dim_{\mathbf{C}} A - \dim_{\mathbf{C}} B - 1\}$.

There are also analagous results (3.1–3.5) for products of Grassmannians that specialize in the case of $\mathbf{P}^N_{\mathbf{C}}$ to Barth's original theorems [3].

In §4 I discuss various generalizations of the results of §3.

I would like to express my thanks to the late H. C. Wang who suggested that I prove my results for non-compact homogeneous complex manifolds, and not only for homogeneous projective manifolds.

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