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A GENERALIZATION OF THE CHEVALLEY RESTRICTION THEOREM

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Introduction. Let $\mathfrak{g}$ be a semisimple Lie algebra over an algebraically closed field k of characteristic zero, let $\mathfrak{h}$ be a Cartan subalgebra of $\mathfrak{g}$, let G be the adjoint group of $\mathfrak{g}$ and let W be the Weyl group of $\mathfrak{g}$ with respect to $\mathfrak{h}$. We denote by $k[\mathfrak{g}]$ (resp. $k[\mathfrak{h}]$) the algebra of polynomial functions on $\mathfrak{g}$ (resp. $\mathfrak{h}$). The Chevalley Restriction Theorem [12, p. 143] states that the restriction homomorphism $k[\mathfrak{g}] \to k[\mathfrak{h}]$ maps $k[\mathfrak{g}]^G$, the algebra of G-invariant polynomials on $\mathfrak{g}$, isomorphically onto $k[\mathfrak{h}]^W$, the algebra of W-invariant polynomials on $\mathfrak{h}$. We will generalize this theorem to the case of a reductive algebraic group G (defined over k) acting on an irreducible normal affine algebraic variety.

We describe our generalization in the case of a reductive group G acting linearly on a vector space M. Assume for simplicity that there exists $y \in M$ such that the orbit $G \cdot y$ is closed and such that $\dim G \cdot y \geqslant \dim G \cdot x$ for every $x \in M$. Then it follows from [7, 10] that there exists a non-empty open subset U of M such that all points of U are of the same orbit type and such that, if $x \in U$, the orbit $G \cdot x$ is closed in M. Let $a \in U$, let H denote the isotropy subgroup G_a, let $N_G(H)$ be the normalizer of H in G and let $W = N_G(H)/H$. Let $F = M^H$ be the vector space of fixed points of H on M. Then our theorem states that the restriction map $k[M] \to k[F]$ maps $k[M]^G$ isomorphically onto $k[F]^W$. (In the case of the adjoint action of G on $\mathfrak{g}$, F is a Cartan subalgebra of $\mathfrak{g}$ and W is the corresponding Weyl group.)

Our generalization of Chevalley's theorem seems to be quite useful in making explicit computations of algebras of invariants for linear representations of reductive groups. Several examples of such computations are given in §6.

A brief sketch of our proof was given in [8]. The proof relies heavily on the existence of an "étale slice" in a neighbourhood of a closed orbit for a reductive group acting on an affine variety [7].

§1. Preliminaries. All algebraic varieties will be taken over the algebraically closed field k of characteristic zero. Our basic reference for algebraic groups and algebraic geometry is [1] and we shall follow the notation and terminology therein. All algebraic groups considered will be affine algebraic groups. If the algebraic group G acts morphically on the algebraic variety X, we say that X is a G-variety. A *G-morphism* $\varphi : X \to Y$ of G-varieties X and Y is a morphism which commutes with the action of G. We say that the (not necessarily connected) algebraic group G is *reductive* if the identity component G^0 is reductive.

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