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THE EQUATION OF A PLANE CURVE

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Introduction. In this paper we continue our study of Petri's theory [2]. Specifically we focus our attention on a beautiful parenthetical remark that Petri makes about the adjoint curves to a plane irreducible curve ([9] p. 191).

In the first section we introduce Petri's invariants τ_j's and σ_j's. The significance of these invariants is the following. Let $\Gamma \subset \mathbf{P}^2$ be a plane irreducible curve with canonical divisor K and line section D and assume that $\dim|D| = 2$. Let d be the maximum integer such that $H^0(K - dD) \neq 0$. Consider the natural maps

$$v_j : H^0(K + (j-1-d)D) \otimes H^0(D) \to H^0(K + (j-d)D).$$

Essentially the invariants τ_j and σ_j measure, respectively, the dimensions of Coker v_j and Ker v_j.

There are very simple formulas linking these invariants with the genus and the degree of Γ. These formulas are given at the end of the first section.

In the second section we show that the equation of the plane curve Γ can be expressed as

$$\det M = 0$$

where M is a matrix of forms, in the homogeneous coordinates X_1, X_2, X_3 of $\mathbf{P}^2$, of the following type.

$$
\begin{array}{rc}
\begin{matrix} \sigma_1\ \{ \\ \sigma_2\ \{ \\ \\ \sigma_{d+1}\ \{ \\ 1\ \{ \end{matrix} &
\begin{bmatrix}
[1] & 0 & 0 & \cdots & 0 \\
[2] & [1] & 0 & \cdots & 0 \\
\vdots & \vdots & & & \vdots \\
[d+1] & [d] & \cdots & & [1] \\
[d+3] & [d+2] & \cdots & & [3]
\end{bmatrix} \\
& \begin{matrix} \underbrace{}_{\tau_0} & \underbrace{}_{\tau_1} & & & \underbrace{}_{\tau_d} \end{matrix}
\end{array}
$$

This symbolism should be understood as follows. The matrix M is expressed in a block form. The size of each block is indicated on the left side and on the bottom of the matrix M. A block labeled by a number ρ consists of forms of degree ρ in X_1, X_2, X_3.

It turns out that the last row of the matrix M plays a privileged role. In fact the minors relative to the elements of this row generate the *adjoint ideal* of Γ, so

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