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THE BOUNDARY OF CLASSICAL SCHOTTKY SPACE

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1. Introduction. On the Riemann sphere, $\mathbf{C} \cup \{\infty\}$, consider a collection of $2g$ mutually disjoint Jordan curves $\alpha_1, \beta_1, \ldots, \alpha_g, \beta_g$ which bound a connected set F. Suppose there exist Möbius transformations A_i, $1 \leqslant i \leqslant g$, with the property that A_i maps the exterior of α_i (that component containing F) onto the interior of β_i. Then the group $G = \langle A_1, \ldots, A_g \rangle$ is free of rank g, purely loxodromic and operates discontinuously on an open, everywhere dense subset $\Omega(G)$ of $\mathbf{C} \cup \{\infty\}$. Furthermore, the region F serves as a fundamental set for this action. Thus, when equivalent boundary points are identified, F represents the quotient surface $\Omega(G)/G$. It is a closed surface of genus g. The complement of $\Omega(G)$ is the limit set of G. It will be denoted by $\Lambda(G)$. We will refer to the generators A_i as corresponding to the pairs of curves (α_i, β_i).

A group which can be generated in such a manner is called a Schottky group. Conversely, every finitely generated group of Möbius transformations which is free, purely loxodromic and discontinuous somewhere on the Riemann sphere is a Schottky group [6]. Chuckrow [3] has shown that for a Schottky group, every set of free generators corresponds to some set of pairs of curves as described above.

For any group of Möbius transformations the action can be extended to the upper half sphere $\mathcal{H}$ of $\mathbf{R}^3$ in a natural manner. It will be discontinuous if and only if the group is discrete (as a matrix group). In the case of a Schottky group G of rank g, the quotient manifold $\mathcal{H} \cup \Omega(G)/G$ is a handlebody of genus g.

We recall that a *geometrically finite* group is one which has a finite sided Poincaré fundamental polyhedron. Equivalently, a geometrically finite group is one whose associated 3-manifold has a natural compactification [5]. Alternatively, such groups can be characterized in terms of properties of points of $\Lambda(G)$ [2]. Using either criterion, Schottky groups are easily seen to be geometrically finite.

A Schottky group is called a *classical Schottky group* if there is some set of generators $\{A_i\}$ for which a corresponding set of curves $\{\alpha_i, \beta_i\}$ can be taken to be *circles* (on the Riemann sphere). Not every Schottky group is classical. This was shown in [4] and, by example, in [7].

Denote by $\mathbf{P}_3$ the complex projective 3-space, and denote by X_g the variety in $\mathbf{P}_3^g$ determined by the equation

$$\prod_{i=1}^{g} \det p_i = 0,$$

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