Vol. 46, No. 2 DUKE MATHEMATICAL JOURNAL © June 1979

SIMULTANEOUS RESOLUTION AND DISCRIMINANTAL LOCI

JONATHAN M. WAHL

Contents

Introduction. Let Spec R be a two-dimensional normal singularity over an algebraically closed field k of characteristic 0, and let $X \to \operatorname{Spec} R$ be the minimal resolution. It is known ([24], [32]) that a deformation of X blows down to a deformation of R iff $h^1(\mathcal{O}_X)$ remains constant; this condition defines a subscheme B of D_X, the deformation space of X. B has been studied by Artin-Schlessinger [2] (who call it Res), where an algebraic convergence theorem is proved. Via the (finite) blowing-down map $\Phi : B \to D_R$ (D_R = deformation space of Spec R), they view B as giving deformations of R which resolve simultaneously, after base change. The aim of this paper is to study the properties of B, the base change given by Φ, and the image $\Phi(B) \subset D_R$. An ultimate goal should be to understand singularities corresponding to generic points of $\Phi(B)$. Our major results, discussed below, are Theorems 2.12, 5.6, and 6.14.

The basic examples are the rational double points (RDP's), denoted by A_n, D_n, E_6, E_7, and E_8. Brieskorn has shown [6] that Φ is surjective, and in fact "is" the map $\mathbf{C}^n \to \mathbf{C}^n / W$, where W = Weyl group of the corresponding Lie algebra. This has been generalized ([2], [17], [34]) to arbitrary rational singularities: $\Phi(B)$ is a smooth irreducible component A (= Artin component) of D_R, and $B \to A$ is Galois, with group a direct product of Weyl groups which can be read off from the graph of R. In the rational case, $B = D_X$ (hence is smooth), and $\Phi(B)$ contains smoothings. If $h^1(\mathcal{O}_X) > 0$, $\Phi(B)$ will be part of the discriminant locus

Received July 14, 1978. Revision received November 17, 1978. Partially supported by NSF grant #MCS77-05724.