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FUNDAMENTAL SOLUTIONS IN COMPLEX ANALYSIS PART II. THE INDUCED CAUCHY RIEMANN OPERATOR

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In Part I of this paper we developed a machinery for constructing fundamental solutions for the Cauchy-Riemann Operator. These same kernels can be used to analyse the induced Cauchy Riemann operator, and we take up this subject here. This approach was pioneered by Henkin-Romanov [HR] and Skoda [SK] for strictly pseudoconvex boundaries. The results here apply to a class of weakly pseudoconvex manifolds which is general enough to include all convex boundaries (including a hyperplane) as well as strictly pseudoconvex boundaries.

In section 7, after briefly discussing the induced Cauchy-Riemann complex, we discuss the boundary behavior of the Bochner-Martinelli transform. Specifically, let $M \subset \Omega \subset \mathbf{C}^n$ be a real hypersurface. Then for $f \in \mathfrak{D}^{p,q}(M)$, the Bochner-Martinelli transform of f is

$$-\int_{\zeta \in M} [B(\zeta, z) \wedge f(\zeta)]^{n, n-1}. \tag{1}$$

The transform is easily seen to be differentiable in $\Omega - M$. We show that it is actually infinitely differentiable up to M from each side. Furthermore, we show, with $B^{\pm}(f)$ defined to be the complex tangential piece of the boundary values of (1) from above and below, that

$$B^{\pm}(f) = B_b(f) \pm \tfrac{1}{2} f, \tag{2}$$

where $B_b(f)$ is the tangential piece of the Cauchy principal value integral

$$\lim_{\epsilon \to 0^+} \int_{\substack{\zeta \in M \\ |\zeta - z| > \epsilon}} B(\zeta, z) \wedge f(\zeta).$$

For $n = 1$, (2) reduces to the Plemelj jump relations for the Cauchy transform.

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