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FUNDAMENTAL SOLUTIONS IN COMPLEX ANALYSIS PART I. THE CAUCHY RIEMANN OPERATOR

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1. Introduction. The method of a priori estimates in several complex variables flourished during the 50's and 60's with the work of Andreotti, Calabi, Hörmander, Kodaira, Kohn, Nakano, and Vessentini. This method is geometrically far reaching, but somewhat restricted to L^2. However, starting with the ground breaking papers of Henkin [HE_1], [HE_2] and Ramirez [R] a more elementary and constructive approach to the Cauchy Riemann equations involving specific kernels has received wide attention. The main purpose of this paper is to develop a machinery for constructing and manipulating these kernels which puts this "kernel approach" on the same footing that the Cauchy-kernel enjoys in one complex variable (or the fundamental solutions E for any other $P(D)$ on $\mathbb{R}^m$). Technically the difficulty is to make sense out of the kernels involved as (Cauchy Principal Value) currents. In addition, we have found some new kernels and applications.

In section 2 we discuss the concept of a fundamental solution for the overdetermined system $\bar\partial$. The development is analogous to the standard development for a single differential operator P. First, the correspondence between operators on functions and kernels which is given by the Schwartz kernel theorem, is extended—with functions replaced by forms. Recall that the operator equation $P\circ E = \mathrm{id}$ and the differential equation $P_x E(y, x) = \delta(x - y)$ are equivalent. (Similarly, $E\circ P = \mathrm{id}$ and ${}^tP_y E(x, y) = \delta(x - y)$ are equivalent.) Of course, with P replaced by $\bar\partial$, there can be no left or right inverse. The substitute for the equations $P\circ E = E\circ P = \mathrm{id}$ is the *homotopy equation* $\bar\partial\circ E + E\circ\bar\partial = \mathrm{id}$. This operator equation corresponds to the differential equation

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