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PHASE SPACE ANALYSIS OF SIMPLE SCATTERING SYSTEMS: EXTENSIONS OF SOME WORK OF ENSS

BARRY SIMON

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§1. **Introduction.** In recent years, the spectral and scattering theory of partial differential operators on $\mathbb{R}^\nu$ has been an extensively studied subject, especially for operators which resemble Schrödinger operators; see [28, 29] for a comprehensive review. Although scattering is basically a time-dependent phenomenon, very few results have been obtained with time-dependent methods. Indeed, the main time-dependent technique is Cook's method [28, §XI.3] which traditionally yields only existence of wave operators. There are certain methods, most notably the Kato-Birman (trace class) theory [28, §XI.3] and the theory of smooth perturbations [29, §XIII.7] which have both time-dependent and time-independent versions but the sharpest results have seemed to require time-independent methods, most notably the Agmon-Kuroda analysis of weighted L^2 estimates [29, §XIII.8].

This situation has been dramatically changed by an exciting and beautiful paper of Enss [13] who uses purely time-dependent methods to obtain virtually identical results to those of the Agmon-Kuroda theory in the case $H = -\Delta + V$ with V a multiplication operator. Particularly exciting developments suggested by Enss' tour de force involve the inclusion of Coulomb potentials and the extension to multiparticle systems. Enss [14] has solved the first of these and has made substantial progress on the second! In this paper, we want to explore the more straightforward extension of studying $H = H_0 + V$ for more general H_0 and V where V is still "localized" in a bounded region of space and H_0 is still an operator with "no scattering" in a geometric sense. It is hoped that these

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