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CROSSED PRODUCTS OF UHF ALGEBRAS BY PRODUCT TYPE ACTIONS

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Abstract. We study the primitive ideal spectrum of the C^*-crossed product of a UHF algebra by certain product type actions of locally compact abelian groups. Any second countable group can be represented in this way with Γ-spectrum equal to the dual group, i.e. such that the crossed product is primitive, but the crossed product is never simple if the group has a nontrivial connected group as a direct summand, it may or may not be simple if the group is totally disconnected non-discrete, and it is always simple if the group is discrete.

1. Introduction. Let $(\mathfrak{A}, G, \alpha)$ be a C^*-dynamical system, where G is a locally compact abelian group, and α is a strongly continuous representation of G in the group of $*$-automorphisms of the C^*-algebra $\mathfrak{A}$. Let $C^*(\mathfrak{A}, G, \alpha)$ be the C^*-crossed product of $\mathfrak{A}$ by G, [12], [29]. The first step in describing the structure of $C^*(\mathfrak{A}, G, \alpha)$ is to investigate its primitive ideal structure. Two aspects of the dynamics α are important for this study, ergodicity and freeness, [13], [14], [20], [28], [29], [33], [34], [35].

Most authors agree that the relevant notion of ergodicity in this context is the ergodicity of the action induced by α on the primitive ideal space $\mathrm{Prim}(\mathfrak{A})$ of $\mathfrak{A}$. It is not clear, however, which notion of freeness of the action should be used. Traditionally, one considers freeness of the induced action on Prim $\mathfrak{A}$, $\hat{\mathfrak{A}}$, etc., or appropriate conditions of "outerness" of the action. Recently it has become clear that a more relevant notion of freeness is associated with the Connes' spectrum $\Gamma(\alpha)$ of the action α, [7], [21]. The main purpose of the present note is to show that this notion of freeness is not strong enough to give a complete characterization of the primitive ideal spectrum of $C^*(\mathfrak{A}, G, \alpha)$, at least if $\mathfrak{A}$ is not type I or G is not discrete.

Let $\hat{G}$ denote the dual group of G, [26], and $\mathrm{Sp}(\alpha) \subseteq \hat{G}$ the (Arveson) spectrum of α, [1]. The Γ-spectrum (= Connes spectrum) $\Gamma(\alpha)$ of α is then defined by

$$\Gamma(\alpha) = \bigcap_{\mathfrak{B}} \mathrm{Sp}(\alpha \mid \mathfrak{B})$$

where $\mathfrak{B}$ ranges over all non-zero α-invariant hereditary subalgebras of $\mathfrak{A}$.

If $(\mathfrak{M}, G, \alpha)$ is a W^*-dynamical system, where $\mathfrak{M}$ is σ-finite and G separable abelian, it is known that the von Neumann crossed product $W^*(\mathfrak{M}, G, \alpha)$ is a factor if and only if $\Gamma(\alpha) = \hat{G}$ and α is ergodic on the center of $\mathfrak{M}$, ([8], Corollary 3.4). Recently, corresponding results have been derived for C^*-dynamical systems, [22], [16]. Recall that an ideal (which in this paper always means a

This is a corrected and extended version of the papers [5], [6].

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