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THE ATOMIC DECOMPOSITION FOR HARDY SPACES IN SEVERAL COMPLEX VARIABLES

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Section 1. Introduction.

Recently characterizations of the classical Hardy spaces have been given which have obvious extensions to very general contexts. These characterizations are in terms of maximal functions [1], [9] and in terms of the atomic decomposition [4], [6]. The purpose of this paper is to study the relationships between these spaces in the context of the unit ball B^n in complex n-space. In fact, we will show that the atomic Hardy spaces and the maximal function Hardy spaces are the same for $0 < p \leq 1$. In addition, we will show the usual Hardy spaces $\mathscr{H}^p$ of holomorphic functions in G^y are the spaces of "holomorphic parts" of functions in the atomic Hardy spaces. These results extend to $0 < p < 1$ the corresponding results for $p = 1$ due to Coifman, Rochberg and G. Weiss [5] and to L. Carleson [2], [3]. As a consequence of these results we obtain a characterization of the dual space of $\mathscr{H}^p$, and we derive an analogue of the factorization theorem of [5] for $\mathscr{H}^p$ $(0 < p \leq 1)$.

Let us be more specific. We denote by $\mathscr{H}^p(B^n)$ $(0 < p < \infty)$ the space of functions $F(z)$ which are holomorphic in the unit ball $B^n = \{z = (z_1, \cdots, z_n) \in \mathbb{C}^n : |z| > 1\}$ and which satisfy

$$\|F\|_{\mathscr{H}^p}^p = \sup_{0<r<1} \int_\Sigma |F(rz)|^p d\sigma(z) < \infty,$$

where $\delta\sigma$ is surface area measure on $\Sigma = \Sigma_{2n-1} = \partial B^n$. By identifying $F \in \mathscr{H}^p$ with its boundary values we identify $\mathscr{H}^z$ as a subspace of $L^p(\Sigma)$.

The space $\mathscr{H}^2$ is of special interest because it is a Hilbert space. Let $P : L^2 \to \mathscr{H}^2$ be the orthogonal projection. Then P has a representation in terms of the Cauchy-Szëgo integral as follows: If $F \in L^2(\Sigma)$, then

$$SF(rz) = c\int_\Sigma \frac{F(\zeta)}{(1 - rz \cdot \bar{\zeta})^n}\, d\sigma(\zeta) \tag{1.1}$$

and

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