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INFINITE REGULAR COVERINGS

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§1. Introduction. In this paper we reconsider the original problem of H. Hopf which served as a starting point for the theory of ends of abstract finitely generated groups: find conditions on a noncompact manifold to be a regular covering space of a *compact* manifold. Both for retaining generality and wider applications one should also consider the problem: find conditions on a noncompact manifold to admit an *infinite*, properly discontinuous group of homeomorphisms. In general, the structure of infinite regular covering spaces can be quite complicated. However, there is a significant class of such spaces which arise in differential geometry and which satisfy some stability or finiteness restrictions. For instance a noncompact connected homogeneous space—which is a good candidate for being an infinite regular covering space—often has *finitely generated* homology; a discrete, torsion free subgroup of $GL_n(\mathbb{R})$ has finite cohomological dimension etc. It turns out that these types of restrictions lead to suprisingly strong conclusions. A typical example of such previously known results is the duality theorem of John Milnor on infinite cyclic coverings (cf. [14] §4.)

Here is a brief summary of the results in this paper. §2 states the basic results of Hopf relating ends of spaces and ends of groups and our extensions of these results to the case when the base space is noncompact. The cohomological formulation of ends is briefly reviewed. §3 studies the case of a space Ω with a single end ϵ at which it is π_1-stable i.e., roughly speaking "the fundamental group at infinity", $\pi_1(\epsilon)$, can be defined. Comparison of $\pi_1(\epsilon)$ and $\pi_1(\Omega)$ in case Ω is an infinite regular covering space leads to some striking results e.g., if $\pi_1(\epsilon) = 1$ then $\pi_1(\Omega) = 1$. §4 studies the cohomology ring with compact supports showing that the multiplication is trivial in case some finiteness restrictions hold. These considerations lead to the following notable result.

THEOREM 4.3 *Let Ω be a connected, paracompact n-dimensional manifold with a single π_1-stable end ϵ. Assume that*

1) $\pi_1(\epsilon) = 1$.

2) $H_*(\Omega; \mathbb{Z})$ *is finitely generated.*

3) $H^*_E(\Omega; \mathbb{Z}) \approx H^*(S^{n-1}; \mathbb{Z})$(i.e., *"infinity is a cohomology $(n-1)$-sphere") and*

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