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BIHOLOMORPHIC MAPS OF WEAKLY PSEUDOCONVEX DOMAINS

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1. Introduction. Boundary regularity of holomorphic mappings between pseudoconvex domains with smooth boundary has been studied by several authors (see [1], [3], [5], [6], [8], [10], and [12]-[18]).

Here we prove the following result.

MAIN THEOREM: *Let Ω_1 and Ω_2 be bounded pseudoconvex domains in $\mathbb{C}^2$ with smooth real analytic boundaries. Then any biholomorphic map $F : \Omega_1 \to \Omega_2$ extends to a homeomorphism $\bar{F} : \bar{\Omega}_1 \to \bar{\Omega}_2$.*

The domains Ω_1 and Ω_2 are not assumed to be strongly pseudoconvex.

We use the same general techniques as were used by Vormoor [16], Henkin [8], Pinčuk [13] and Range [15]. Hence the principal difficulty is to estimate the Carathéodory metric of Ω_2. To do this we use the peak functions obtained in [2]. The Main Theorem then is an immediate corollary of

THEOREM 1. *Let M_1, M_2 be 2-dimensional Stein manifolds, and let $\Omega_1 \subset M_1$, $\Omega_2 \subset M_2$ be relatively compact pseudoconvex domains with $\mathscr{C}^2$ boundary. If $\partial\Omega_2$ is real analytic, and if $F : \Omega_1 \to \Omega_2$ is a proper holomorphic mapping, then $F \in \mathrm{Lip}^{\epsilon}(\bar{\Omega}_1)$ for some $\epsilon > 0$.*

It would be desirable to extend this Theorem to $\mathbb{C}^n$ for $n > 2$, but our proof does not apply to that case. The principal difficulty is that we do not know how to construct peak functions for each $p \in \partial\Omega_2$.

In Section 2 we estimate the Carathéodory metric at a strongly pseudoconvex boundary point in terms of the distance to the weakly pseudoconvex points. Then in Section 3 we estimate the Carathéodory metric on certain models of weakly pseudoconvex sets, and in Section 4 we obtain global estimates on the metric by combining the results in the previous two sections. The estimates are proved only in $\mathbb{C}^2$, but the proofs and results extend to Stein manifolds. Finally, in Section 5, we explain how Theorem 1 is obtained from section 4.

2. Estimate of the Carathéodory metric at a strongly pseudoconvex point. We let $\Omega \subset \mathbb{C}^2$ be a bounded pseudoconvex domain with real analytic boundary, and we define for $z = (z_1, z_2) \in \Omega$ and $\xi = (\xi_1, \xi_2) \in \mathbb{C}^2$,

$$(2.1) \quad c(z; \xi) = \sup \left\{ \left| \sum_{j=1}^{2} \frac{\partial f}{\partial z_j}(z)\xi_j \right| ; f : \Omega \to \mathbb{C} \text{ is holomorphic and } |f| < 1 \right\}$$

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