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REMARKS ON THE FILLING SCHEME FOR RECURRENT MARKOV CHAINS

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One of the goals of "filling scheme" procedures is to give an explicit construction of reduced measures or, equivalently, of minimal positive solutions to some Poisson equations. More specifically, let P be the transition probability of a Markov chain and let (λ, μ) be a pair of probability measures on the state space $(E, \mathscr{E})$. The filling scheme associates with (λ, μ) the sequence of pairs of measures (λ_n, μ_n) defined inductively by

$$\lambda_0 = (\lambda - \mu)^+, \qquad \mu_0 = (\lambda - \mu)^-,$$

$$\lambda_n = (\lambda_{n-1}P - \mu_{n-1})^+, \qquad \mu_n = (\lambda_{n-1}P - \mu_{n-1})^-,$$

(see [1], [2], [4], [9], [10]). The sequence (μ_n) is easily seen to decrease to a measure μ_∞. Define

$$\xi_n = \sum_0^n \lambda_k$$

and

$$\xi = \sum_0^\infty \lambda_k;$$

from the identity $\lambda_n - \mu_n = \lambda_{n-1}P - \mu_{n-1}$, it follows that

$$\xi_{n+1} = \xi_n P + (\lambda - \mu) + \mu_{n+1},$$

and passing to the limit in this equation yields

$$\xi = \xi P + (\lambda - \mu) + \mu_\infty.$$

If $\mu_\infty = 0$, the measure ξ is a positive solution of the Poisson equation $\xi = \xi P + \lambda - \mu$; but there is no reason why ξ should be σ-finite. If the potential kernel $G = \sum_0^\infty P_n$ is proper, then $\xi \leq \lambda G$, hence ξ is σ-finite. In this paper we want to investigate to what extent this is still true for recurrent Markov chains. The hypotheses that ξ is σ-finite occurs in some papers (see for example [11] p. 13) and was sometimes tacitly assumed.

The above question ties in with a result of Baxter and Chacon. In [2] they study aperiodic Markov chains with quasi-compact transition probabilities and

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