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$\mathscr{C}^\infty$ PEAK FUNCTIONS FOR PSEUDOCONVEX DOMAINS OF STRICT TYPE

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Introduction

Let U be a bounded pseudoconvex domain in $\mathbb{C}^n$ with smooth boundary M. We let $A^k(\bar{U})$ denote the algebra of functions analytic on U and of class $\mathscr{C}^k$ on $\bar{U}$, $0 \leq k \leq +\infty$.

Given a point $P \in M$ does there exist a function $F \in A^\infty(\bar{U})$ such that $F(P) = 1$ and $|F(q)| < 1$ for $q \in \bar{U} - \{P\}$? That is, is P a peak point for the algebra $A^\infty(\bar{U})$?

Hakim and Sibony [8, 9] and P. Pflug [13] have shown how to use the work of J. J. Kohn [12] to reduce the global problem to a local one. Given $P \in M$ does there exist a neighborhood V of P and a function $F \in A^\infty(V \cap \bar{U})$ such that $F(P) = 1$ and $|F(q)| < 1$ for $q \in V \cap \bar{U} - \{P\}$?

Fornaess has shown [6] that if M in $\mathbb{C}^2$ is locally defined by

$$\mathrm{Re}(w) + |z|^6 + t|z|^2\mathrm{Re}(z^4) = 0,$$

where $1 < t < (9/5)$ there is no function in $A^1(\bar{U})$ which peaks at $P = (0, 0)$. This example has the same geometric property as an earlier example of Kohn-Nirenberg [10], namely, there is no holomorphic support function at P.

Hakim and Sibony [9] and also Range [15] have shown that if there is a holomorphic $(n - 1)$-dimensional support manifold intersecting M only at P then there exists an $F \in A^0(V \cap \bar{U})$ which peaks at P. Hakim and Sibony [9] also show that if P is of "strict type" (see 1.6—this is termed condition S_3 in this paper) there is a function $F \in A^1(V \cap \bar{U})$ which peaks at P.

The result of Hakim and Sibony does not use the pseudoconvexity of M. In this paper we will prove (Theorem 3.8) that if M is pseudoconvex and P satisfies a "strict type" condition—termed condition S_2 in this paper (it is a stronger condition than the one used by Sibony and Hakim) then there is a function F holomorphic in a neighborhood of P in $\mathbb{C}^n$ such that the maximum of $|F|$ on $V \cap \bar{U}$ is attained only at P. We will show that the result is not true, in general, if P satisfies only condition S_3. In the example of 4.2, P satisfies condition S_3 but there is no $F \in A^\infty(V \cap \bar{U})$ which peaks at P. Now condition S_3 implies there is a holomorphic support manifold (see 1.10). Thus we have given an example of a point on a pseudoconvex set U with smooth boundary where

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