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ADDENDUM TO THE ARTICLE "ON THE TORSION IN $K_*(\mathbb{Z})$"

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We shall use the notations and references of the article of R. Lee and R. H. Szczarba, which will be referred to as [LS]. (Duke Math. J. **45**(1978), 101–129.

1. THEOREM 1. *The groups $K_4(\mathbb{Z})$ and $K_5(\mathbb{Z})$ contain no 5-torsion.*

This result is a consequence, via Lemma 1.1 and the exact sequences of Quillen (paragraph 1 and Proposition 5.1), of the following:

THEOREM 1′. *Modulo 2 and 3-torsion, we have*

$$H_1(SL_4(\mathbb{Z}) \cdot St_4) = H_1(GL_5(\mathbb{Z}), St_5) = 0.$$

$$H_2(SL_4(\mathbb{Z}), St_4) = 0.$$

2. LEMMA. *Let n be an integer greater than one. The boundary ∂X_n^* of X_n^* has the homotopy type of the Tits' building of the parabolic subgroups of $SL_n(\mathbb{Q})$ and X_n^* is contractible.*

Proof. As in [LS], §2, one gives the CW-topology to X_n^*.

Let $C(W)$ be the set of positive semi-definite real quadratic forms h with kernel generated by W, a subspace of $V = \mathbb{Q}^n$, modulo scalars. Following [13], ∂X_n^* is the (disjoint) union of the $C(W)$'s for all proper non zero subspaces W of V. Furthermore, in ∂X_n^*, we have $\overline{C(W)} = \coprod_{W' \supset W} C(W')$.

Let T_n be (the geometric realisation of) the Tits building of $SL_n(\mathbb{Q})$, i.e., the nerve of the set of proper non zero subspaces of V, ordered by inclusion, and let $T_W \subset T_n$ be the nerve of the set of proper subspaces of V containing W. Clearly T_W is homotopically trivial.

The subcomplex $\overline{C(W)}$ of X_n^* is contractible. To prove this we remark that if A_0 and A_1 are two points in C_n^*, one can choose finitely many cells covering all the segments $[A_0, A]$, for A close enough to A_1 (in the CW-topology). We then use the convexity of $\overline{C(W)}$.

Using a barycentric subdivision of X_n^*, one can see that $\overline{C(W)}$ is an absolute retract, and, similarly to [4], §8, this allows us to construct a map f from ∂X_n^* to T_n such that $f(\overline{C(W)}) \subset T_W$ (use an induction process on the codimension of W). We then construct a map $g : T_n \to X_n$ such that $g(T_W) \subset \overline{C(W)}$, i.e., $g((W_1 \subset W_2 \subset \cdots \subset W_k)) \subset \overline{C(W_1)}$, and prove that g is an homotopic inverse for f as in loc. cit. This homotopy equivalence commutes, up to homotopy, to the action of $SL_n(\mathbb{Z})$. q.e.d

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