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ACYCLIC MODELS FOR MULTICOMPLEXES

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Introduction

The classical method of acyclic models, due to Eilenberg-MacLane [8], has proved a very powerful tool in constructing chain-mappings and homotopies, comparing different homology theories, etc. Our purpose here is to extend the method of acyclic models to bicomplexes; the method will then enable us to construct mappings with good filtration properties, thus yielding mappings of spectral sequences.

It turns out, however, that the suitable notion is *not* that of a bicomplex, but that the proper setting is the theory of multicomplexes, introduced by Wall [19], and exploited by Liulevicius [13]. Indeed, a secondary aim of this paper is to provide propaganda for the category of multicomplexes. Cartier and Dold [4], [5] have defined the chain-complex Hom(A, B), where A, B are chain-complexes, and shown its utility in discussing acyclic models. Similarly, we define a multicomplex $\mathcal{H}om(X, Y)$, where X, Y are multicomplexes; the study of the spectral sequence of $\mathcal{H}om(X, Y)$ then yields very easily our acyclic models theorem for multicomplexes.

We apply the technique to bisimplicial sets and cosimplicial simplicial sets, obtaining generalized Eilenberg-Zilber theorems, and we construct Steenrod reduced powers, for an arbitrary prime p, in the cohomology spectral sequence of a bisimplicial (or cosimplicial simplicial) module with commutative and associative diagonal. Our methods are very natural, being straightforward generalizations of the standard techniques for simplicial modules; the notion of multicomplex morphism enables us to keep track automatically of the filtrations involved. In the bisimplicial case, we obtain Steenrod operations of "base type" and of "fiber type"; for $p = 2$, this has already been done by Singer [15], [16]. In the cosimplicial simplicial case, we obtain Steenrod operations of "base type", thus answering in the affirmative a question of Singer [15], and completing the results of Rector [14], and of Smith [17], who constructed "fiber type" operations in the special case of the Eilenberg-Moore spectral sequence.

It seems appropriate here to express our three-fold gratitude to Liulevicius: (1) for introducing us in [12] to the beautiful Dress approach [7] to the Serre spectral sequence, (2) for making somewhat cryptic statements concerning the multiplicative structure of the Serre spectral sequence ([12], p. 96, "proof" of theorem 5.12), which aroused our interest in the subject, and finally, (3) for providing in [13] the tools suitable for a solution of the problem.

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