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MULTIPLICATION BY THE COORDINATE FUNCTIONS ON THE HARDY SPACE OF THE UNIT SPHERE IN $\mathbb{C}^n$

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1. Introduction. Let B^n be the unit ball and S^n the unit sphere in n-dimensional complex Euclidean space. Let σ denote surface area measure on S^n and write $L^\infty(S^n)$ for $L^\infty(\sigma)$, $L^2(S^n)$ for $L^2(\sigma)$ etc. $H^2(S^n)$ denotes the closure in $L^2(S^n)$ of the polynomials in the coordinate functions $z_1, \cdots, z_n$. $C(S^n)$ denotes the algebra of continuous functions on S^n. If $f \in L^\infty(S^n)$ then the Poisson integral of f gives a bounded harmonic function F on B^n and F has radial boundary limits equal to f almost everywhere (see [21]). This correspondence gives an isometry between $L^\infty(S^n)$ and the space of bounded harmonic functions on B^n with the supremum norm. Under this correspondence, the algebra of bounded analytic functions on B^n corresponds to a closed subalgebra $H^\infty(S^n)$ of $L^\infty(S^n)$. Also, for $f \in H^2(S^n)$, the Poisson integral of f defines a holomorphic function in B^n which we also denote by f. We shall have occasion to use this extension of f and shall do so without further comment.

If $\phi \in L^\infty(S^n)$ we denote by T_ϕ the operator on the Hilbert space $H^2(S^n)$ defined by $T_\phi f = P(\phi f)$ where P denotes the orthogonal projection of $L^2(S^n)$ on $H^2(S^n)$. T_ϕ is called the Toeplitz operator with symbol ϕ. Of course when $\phi \in H^\infty(S^n)$ the action of P is redundant since $\phi H^2(S^n) \subseteq H^2(S^n)$. We write T_i for T_{z_i} $(1 \leq i \leq n)$ where z_i is the i^{th} coordinate function. Note that $T_1^*T_1 + \cdots + T_n^*T_n = I$.

For any Hilbert space H, $BL(H)$ will denote the algebra of all bounded linear operators on H. I will denote the identity operator in $BL(H)$.

In the case $n = 1$ there is a vast amount of literature concerning the study of Toeplitz operators (for an account of this theory see Chapter 7 of Douglas' book [9]). In this case T_1 is simply the well-known unilateral shift. This operator is perhaps the most studied of all particular bounded operators on a separable Hilbert space. We refer to [10] for a description of some of the basic results concerning this operator. The most important result concerning the unilateral shift is Beurling's theorem which gives a complete description of the invariant subspaces and cyclic vectors for the operator (see [18]). In this paper we wish to begin an investigation into the properties of the pair of Toeplitz operators $\{T_1, T_2\}$ acting on $H^2(S^2)$. These operators could be considered as a 'spherical' two-variable analogue of the unilateral shift. The 'bidisc' analogue would be the pair $\{T_{z_1}, T_{z_2}\}$ of Toeplitz operators acting on $H^2(T^2)$ where T^2 is the torus in $\mathbb{C}^2$. (Definitions of the Hardy spaces etc. for the torus T^n can be found in [19]). In

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