Vol. 44, No. 2 DUKE MATHEMATICAL JOURNAL © June 1977

ON NORM MAPS FOR ONE DIMENSIONAL FORMAL GROUPS III

MICHIEL HAZEWINKEL

1. Introduction

The main purpose of the present note is to give a more elementary and conceptual and less computational proof of the main theorem of [4]. At the same time we generalize the theorem.

Let K be a local field, L/K a finite galois extension. Let A be the ring of integers of K and $F(X, Y)$ a (commutative one dimensional) formal group (law) over A, i.e. $F(X, Y)$ is a formal power series in two variables over A of the form $F(X, Y) = X + Y + \sum_{i,j\geq 1} a_{ij}X^iY^j$ such that $a_{ij} = a_{ji}$ and $F(F(X, Y), Z) = F(X, F(Y, Z))$. Let $\mathfrak{m}_L$ be the maximal ideal of $A(L)$, the ring of integers of L. The group recipe $F(X, Y)$ can be used to define a new abelian group structure on the set $\mathfrak{m}_L$, viz. $x +_F y = F(x, y)$ where $x, y \in \mathfrak{m}_L$. This group is denoted $F(L)$. There is a natural norm map

$$(1.1.)\quad F - \mathrm{Norm}_{L/K} : F(L) \to F(K), \qquad x \mapsto \sigma_1 x +_F \sigma_2 x +_F + \cdots +_F \sigma_n x$$

where $\{\sigma_1, \cdots, \sigma_n\} = \mathrm{Gal}\,(L/K)$. The general problem is to describe the image (or the cokernel) of the maps $F - \mathrm{Norm}_{L/K}$. For example if F is the multiplicative group $\hat{\mathbf{G}}_m(X, Y) = X + Y + XY$, then $F - \mathrm{Norm}$ becomes the ordinary norm map

$$(1.2)\qquad N_{L/K} : U^1(L) \to U^1(K)$$

where $U^1(L) = \{x \in U(L) = A(L)^* \mid x \equiv 1 \bmod \mathfrak{m}_L\}$. The study of Coker $N_{L/K}$ is what a not inconsiderable part of local class field theory is about.

Let K_∞/K be an infinite galois extension of Galois group isomorphic to $\mathbf{Z}_p$, the p-adic integers. Such an extension is called a $\mathbf{Z}_p$-extension or a Γ-extension. Let K_n be the invariant field of the closed subgroup $p^n\mathbf{Z}_p$, $n = 0, 1, 2, \cdots$, where we write $K_0 = K$.

The abelian group $F(K)$ carries a natural filtration $F(K) = F^1(K) \supset F^2(K) \supset \cdots \supset F^n(K) \supset \cdots$ where $F^n(K) = \{x \in F(K) \mid x \in \mathfrak{m}_K^n\}$.

We write $F - \mathrm{Norm}_{n/0}$ for $F - \mathrm{Norm}_{K_n/K}$. The main theorem of this paper is now.

1.3. THEOREM. *Let K_∞/K be a totally ramified $\mathbf{Z}_p$-extension of an absolutely unramified mixed characteristic local field K with perfect residue field k of characteristic $p > 2$. Let $F(X, Y)$ be a formal group over A of height $h \geq 2$. Then we*

Received December 9, 1975. Revision received October 7, 1976.