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SEMI-DIRECT PRODUCTS OF FUCHSIAN GROUPS AND UNIFORMIZATION

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I. Introduction

1. Uniformization of algebraic varieties.

Using the theory of quasi-Fuchsian groups Griffiths [7] proved that for any quasi-projective variety A_0 of dimension n, there is a Zariski open subset A_1 such that the universal covering of A_1 is a bounded domain in $\mathbf{C}^n$, a so called Bergman domain.

We consider the converse problem here; the approach centers on defining an explicit action for a discontinuous group on a Bergman domain, so as to obtain as quotient a complex manifold, not necessarily algebraic. The techniques used to construct a group of analytic automorphisms as a split extension of two known Fuchsian groups are developed in Chapter 2, where we prove our main theorem.

All the previous results in this note are established in Bers [3] and so we keep the notations used there. It seems pertinent to include in an appendix a correction to a proof in that paper.

2. Teichmüller space and modular groups.

We shall collect in this section a summary of the notations and results which we shall use; for further details, the reader may consult Bers [2].

Let U denote the upper half plane in $\mathbf{C}$, L the lower half plane. A Beltrami differential μ for a Fuchsian group G acting on U, is an element in $L_\infty(U)$ with $\|\mu\|_\infty < 1$ such that

$$\mu(g(z))\,\frac{\overline{g'(z)}}{g'(z)} = \mu(z) \qquad z \in U, \qquad g \in G. \tag{1}$$

We define W^μ (resp. W_μ) as the unique homeomorphic solution of the differential equation

$$\frac{\partial w^\mu}{\partial \bar{z}}(z) = \hat{\mu}(z)\,\frac{\partial w^\mu}{\partial z}(z), \qquad z \in \mathbf{C}, \tag{2}$$

fixing $0, 1, \infty$, where $\hat{\mu}$ is extended to L by 0 (resp. by setting $\hat{\mu}(\bar{z}) = \overline{\hat{\mu}(z)}$, $z \in U$).

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