Vol. 44, No. 1 DUKE MATHEMATICAL JOURNAL© March 1977

ASYMPTOTIC BEHAVIOUR OF MATRIX COEFFICIENTS OF THE DISCRETE SERIES

DRAGAN MILIČIĆ

Introduction. Let G be a connected semisimple Lie group with finite center. The set of equivalence classes of square-integrable representations of G is called the discrete series of G. The discrete series play a crucial role in the representation theory of G, as can be seen from Harish-Chandra's work on the Plancherel formula [8].

In the following we suppose that G has nonempty discrete series. Therefore, by a result of Harish-Chandra, G has a compact Cartan subgroup H. Let K be a maximal compact subgroup of G containing H. The K-finite matrix coefficients of discrete series representations are analytic square-integrable functions on G. Moreover, Harish-Chandra has shown that they decay rapidly at infinity on the group. For various questions of harmonic analysis on G it is useful to have a better knowledge of their rate of decay. This problem was studied by P. C. Trombi and V. S. Varadarajan [16]. They have found a simple necessary condition on a discrete series representation having the K-finite matrix coefficients with certain rate of decay. Our main result is that their condition is sufficient too. In turn, this gives a precise characterization of discrete series representations whose K-finite matrix coefficients lie in $L^p(G)$ for $1 \leq p < 2$.

To describe this result we must recall Harish-Chandra's parametrisation of the discrete series representations. Let $\mathfrak{g}_0$, $\mathfrak{k}_0$ and $\mathfrak{h}_0$ be the Lie algebras of G, K and H, and $\mathfrak{g}$, $\mathfrak{k}$ and $\mathfrak{h}$ their complexifications, respectively. Denote by Φ the root system of $(\mathfrak{g}, \mathfrak{h})$. A root $\alpha \in \Phi$ is called compact if its root subspace is contained in $\mathfrak{k}$ and noncompact otherwise. Let W be the Weyl group of $(\mathfrak{g}, \mathfrak{h})$ and W_k its subgroup generated by the reflections with respect to the compact roots. The Killing form of $\mathfrak{g}$ induces an inner product $(\ |\ )$ on $i\mathfrak{h}_0^*$, the space of all linear forms on $\mathfrak{h}$ which assume imaginary values of $\mathfrak{h}_0$. An element λ of $i\mathfrak{h}_0^*$ is singular if it is orthogonal to at least one root in Φ, and nonsingular otherwise. The differentials of the characters of H form a lattice Λ in $i\mathfrak{h}_0^*$. Let ρ be the half-sum of positive roots in Φ, with respect to some ordering on $i\mathfrak{h}_0^*$. Then $\Lambda + \rho$ does not depend on the choice of this ordering.

Harish-Chandra has shown that to each nonsingular $\lambda \in \Lambda + \rho$ we can attach a class π_λ of discrete series representations, π_λ is equal to π_μ if and only if λ and μ are conjugate under W_k and the discrete series are exhausted in this way [8].

Roughly speaking, our result shows that the rate of decay of K-finite matrix

Received October 7, 1976. Supported in part by NSF grant MPS72-05055 A03 and SIZ VI SRH.