Vol. 43, No. 4 DUKE MATHEMATICAL JOURNAL© December 1976

MOEBIUS-INVARIANT FUNCTION SPACES ON BALLS AND SPHERES

ALEXANDER NAGEL AND WALTER RUDIN

I. Introduction.

Throughout this paper, n denotes a positive integer, and $\mathbf{C}^n$ is the vector space of all ordered n-tuples $z = (z_1, \cdots, z_n)$ of complex numbers z_i, made into a Hilbert space by means of the usual inner product

$$\langle z, w \rangle = z_1\bar{w}_1 + \cdots + z_n\bar{w}_n$$

and the corresponding norm

$$|z| = \langle z, z \rangle^{1/2}.$$

We put

$$B = \{z \in \mathbf{C}^n : |z| < 1\},$$
$$S = \{z \in \mathbf{C}^n : |z| = 1\},$$
$$\bar{B} = B \cup S.$$

Thus B and $\bar{B}$ are the open and closed unit balls of $\mathbf{C}^n$, respectively. Their boundary S is a sphere of (real) dimension $2n - 1$ which carries a (unique) rotation-invariant probability measure σ, defined on the Borel subsets of S. The notation $L^p(S)$, for the usual Lebesgue spaces, refers to this measure σ.

The *Moebius group*, denoted by $\mathfrak{M}$, is the group of all one-to-one holomorphic maps of B onto B. The members of $\mathfrak{M}$ are described in detail in Part II, but let us mention immediately that each $\phi \in \mathfrak{M}$ extends to a homeomorphism of $\bar{B}$ onto $\bar{B}$, and that ϕ therefore carries S to S.

[Note that the dimension n is not explicitly stated in the notations B, S, σ, $\mathfrak{M}$. Since no more than one value of n will occur in any discussion, this simplified notation should cause no confusion.]

A space X of functions with domain $\bar{B}$(or S, or B) will be called a *Moebius space*, or an $\mathfrak{M}$-invariant space, if the composition $f \circ \phi$ belongs to X for every $f \in X$ and every $\phi \in \mathfrak{M}$. The following Moebius spaces will occur:

$C(\bar{B})$: the continuous complex functions on $\bar{B}$.
$C(S)$: the continuous complex functions on S.
$C_0(B)$: those $f \in C(\bar{B})$ that vanish on S.
$A(B)$: those $f \in C(\bar{B})$ that are holomorphic in B.

Received May 17, 1976. Both authors were partially supported by NSF Grant MPS 75-06687.