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A REMARK ON A QUESTION OF MARGULIS

SU-SHING CHEN

1. Introduction.

In [9], Selberg proved that every finitely generated subgroup G of $GL(n, \mathrm{K})$ over a field K of characteristic 0 contains a normal subgroup H of finite index with no elements of finite order. This beautiful theorem is a generalization of a result of Bundgaard-Nielsen-Fox [3], [5] which is known as the Fenchel's conjecture. They proved that every finitely generated Fuchsian group G contains a normal subgroup H of finite index which has no elliptic elements. Their proof was group theoretic and geometric, while Selberg's proof was algebraic. In [8], Margulis asked some unsolved problems in the theory of discrete subgroups of Lie groups. Among them, we are interested in the following problems. Let G be a properly discontinuous group of isometries of a noncompact symmetric space X with compact quotient X/G. Selberg's theorem implies that G contains a normal subgroup H of finite index acting on X without fixed points. Can this fact be proved geometrically? Is the analogous theorem true when X is an arbitrary, simply connected, complete, Riemannian manifold with nonpositive sectional curvature which is not necessarily symmetric?

In this paper, we shall prove the following theorems including an affirmative answer to the second problem of Margulis when the Euclidean factor of the de Rham decomposition of X is trivial and the isometry group $I(X)$ of X or the properly discontinuous group G of isometries satisfies certain conditions. Our proof depends on Selberg's theorem. In fact, the reduction is very simple. One may pursue a purely geometric proof along the lines of Bundgaard-Nielsen-Fox. The major difficulty is that we do not know what kind of relations may occur among a finite set of generators of G.

THEOREM 1. *Let X be a simply connected, complete, Riemannian manifold with nonpositive sectional curvature and without Euclidean factor. Let G be a properly discontinuous group of isometries of X such that X/G is compact and G is contained in the connected isometry group $I_0(X)$. Then G has a normal subgroup H of finite index acting on X without fixed points.*

THEOREM 2. *Let X be a simply connected, complete, Riemannian manifold with nonpositive sectional curvature and without Euclidean factor. Assume that $I(X)/I_0(X)$ is finite. If G is a properly discontinuous group of isometries of X such that X/G is compact, then G has a normal subgroup H of finite index acting on X without fixed points.*

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