Vol. 43, No. 4 DUKE MATHEMATICAL JOURNAL© December 1976

# COMPLETELY POSITIVE MAPS ON C*-ALGEBRAS AND THE LEFT MATRICIAL SPECTRA OF AN OPERATOR

JOHN BUNCE AND NORBERTO SALINAS

**§1. Introduction.** Throughout this paper $\mathcal{H}$ will denote a fixed, separable, infinite dimensional, complex Hilbert space, although not every complex Hilbert space considered here will be separable. Given a complex Hilbert space $\mathcal{G}$, we shall denote by $\mathcal{B}(\mathcal{G})$ the algebra of all (bounded) operators on $\mathcal{G}$ and by $\mathcal{K}(\mathcal{G})$ the ideal of all compact operators on $\mathcal{G}$.

One of our purposes in the present paper is to produce a characterization of the $w^*$-closure of all "spatial" completely positive maps from a separable $C^*$-subalgebra $\mathfrak{A}$ of $\mathcal{B}(\mathcal{H})$ into the algebra of all complex $n \times n$ matrices (cf. §2). This result is the natural generalization to the matricial case of a similar characterization of the weak closure of the set of all vector states of $\mathfrak{A}$ [12]. Secondly, we apply the results of §2 to obtain a spatial characterization of the essential matricial range (cf. §3), the left essential matricial spectrum (cf. §4), and the essential matricial spectrum (cf. §5) of an operator $T$ in $\mathcal{B}(\mathcal{H})$. These sets are the natural matricial generalizations of the essential numerical range, the left essential spectrum and the essential spectrum of the operator $T$, respectively. These are objects canonically associated with the image of the operator $T$ under the natural quotient map $\mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})/\mathcal{K}(\mathcal{H})$ (cf. §3, §4, §5, for the corresponding definitions). As a second by-product of the results of §2 we produce an alternative proof (more in the $C^*$-algebra framework) of the spatial characterization of the reducing essential matricial spectra of an operator, which was given in [16]. Perhaps one of the most interesting consequences presented in this paper of the above-mentioned characterizations is Theorem 5.6—an extension of a result of Bonsall [3] which asserted that if the spatial matricial ranges of two given operators coincide for all dimensions then the essential numerical ranges also coincide.

We may say that our general aim in the present paper is to give some further insight in the important relationship (already noticed by several authors, [1], [2], [10]) between $C^*$-algebra theory and operator theory. This is the reason that in the latter sections of the paper we restrict attention to singly generated $C^*$-algebras.

Finally, although many of our results could be stated and proved for $C^*$-algebras without identity, for simplicity we assume that all our $C^*$-algebras have

Received October 4, 1975. Revision received May 24, 1976.