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# THE ROLES OF NONDENSITY POINTS

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Consider a density $P$ on a hyperbolic Riemann surface $R$, i.e. a nonnegative locally Hölder continuous second order differential $P = P(z)dxdy$ $(z = x + iy)$ on $R$. A point $z^*$ on the Royden harmonic boundary $\Delta$ of $R$ is said to be a *Green energy nondensity* point of $P$ if there exists an open neighborhood $U^*$ of $z^*$ in the Royden compactification $R^*$ of $R$ such that

$$(1) \qquad \int_{(U^*\cap R)\times(U^*\cap R)} G(z, \zeta)P(z)P(\zeta)\, dx\, dy\, d\xi\, d\eta < \infty,$$

where $G(z, \zeta)$ is the harmonic Green's function on $R$. If the condition

$$(2) \qquad \int_{U^*\cap R} P(z)\, dx\, dy < \infty$$

holds, then $z^*$ is called a *nondensity* point of $P$. The set of Green energy nondensity points of $P$ is denoted by $\Delta_P$ and the nondensity points of $P$ by $\Delta^P$. The set $\Delta_P$ ($\Delta^P$, resp.) was introduced in [4] ([1], resp.) for the purpose of studying the space $PD(R)$ ($PE(R)$, resp.) of solutions of the equation $\Delta u = Pu$ with finite Dirichlet integrals

$$D_R(u) = \int_R |\nabla u|^2\, dx\, dy < \infty$$

(finite energy integrals

$$E_R(u) = D_R(u) + \int_R u^2 P < \infty,$$

resp.) The subspaces $PBD(R)$, $PBE(R)$ of bounded functions in $PD(R)$, $PE(R)$ are especially suited for study in terms of their behavior on $\Delta_P$, $\Delta^P$ (cf. [2], [1]). For this reason it is natural to ask the

*QUESTION. Do the sets $\Delta_P$, $\Delta^P$ characterize the spaces $PBD(R)$, $PBE(R)$?*

More precisely, let $Q$ be another density on $R$. We shall discuss whether $\Delta_P = \Delta_Q$($\Delta^P = \Delta^Q$, resp.) is equivalent to the existence of a canonical isomorphism $S$ between $PBD(R)$ and $QBD(R)$ ($PBE(R)$ and $QBE(R)$, resp.). Here a vector space isomorphism $\Psi$ between $PBX(R)$ and $QBX(R)$ is called *canonical* if for every $u \in PBX(R)$ there is a potential $p_u$ on $R$ such that $|u - \Psi u| \le p_u$, $X = D, E$, (cf. [5]). The main conclusion of this paper is that the answer to this question is in general in the negative.

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