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ON DENSITY PROPERTIES OF CERTAIN SUBGROUPS OF LOCALLY COMPACT GROUPS

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Let G be a locally compact group and H a closed subgroup of G such that the quotient space G/H has a finite invariant Haar measure. The finiteness condition on Haar measure implies very strong density properties for H. In [36], algebraic density properties for such subgroups of analytic groups were studied. Here we discuss algebraic and topological density properties of such subgroups of locally compact groups. We generalize some of the results in [36] and add new applications. In section 1, we consider algebraic density properties in terms of representation theory. The following theorem is proved.

THEOREM 1.4. *Let G be a locally compact group and H a closed subgroup such that G/H has a finite invariant Haar measure. Let $\pi : G \to GL(n, \mathbf{R})$ be a continuous representation, and, by an abuse of notation, let G^* and H^* be the Zariski closures of $\pi(G)$, $\pi(H)$ in $GL(n, \mathbf{C})$ respectively, and $G_{\mathbf{R}}^* = (G^*)_{\mathbf{R}}$, $H_{\mathbf{R}}^* = (H^*)_{\mathbf{R}}$. Then we have the following conditions*:

(i) $(G_{\mathbf{R}}^*)^\circ \subset (H_{\mathbf{R}}^*)\pi(G)$.

(ii) $G_{\mathbf{R}}^*/H_{\mathbf{R}}^*$ (resp. $G_{\mathbf{R}}^*/(H_{\mathbf{R}}^*)^\circ$) *is compact and has an invariant Haar measure.*

(iii) $\pi^{-1}(H_{\mathbf{R}}^*)$ *is uniform in* G, i.e. $G/\pi^{-1}(H_{\mathbf{R}}^*)$ *is compact.*

This result generalizes [36, Theorem 3.1]. As an application of this density theorem, we use (iii) of Theorem 1.4 to derive the following theorem on cohomological theory of locally compact groups.

THEOREM 1.6. *Let G be a locally compact group and L a closed subgroup of G such that G/L has a finite invariant Haar measure. Let V be a finite dimensional real vector space and $\pi : G \to GL(V)$ a continuous representation. Then the restriction homomorphism $H^1(G, V) \to H^1(L, V)$ is injective.*

In the case that G is an analytic group and L a uniform lattice of G, this theorem was already presented in [6]. We use the same argument as in [6] for the general case. However one has to prove the summability of the integral used in the proof of [6]. By appealing to (iii) of Theorem 1.4, we can overcome this difficulty.

In section 2, we discuss topological density properties. Theorem 2.4 is the analogue of Mahler's theorem on compactness conditions in the space of lattices. Other compactness and closedness conditions are also studied. In section 3,

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