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A REMARK ON THE UNIVERSAL COVER OF A MOISHEZON SPACE

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The purpose of this note is to prove the following.

THEOREM. *Let M be a compact Moishezon manifold. Suppose $D \xrightarrow{\pi} M$ is a covering of M. If D is a domain in a Stein manifold X then D is Stein. More generally, D need only be required to be a domain spread over an open subset of a Stein manifold X.*

The authors would like to thank Phillip Griffiths for suggesting this problem.

Recall that an n-dimensional compact complex manifold M is Moishezon if there are n functionally independent meromorphic functions on M. The characterization of Moishezon which we shall use is that M is Moishezon if and only if there exists a one-to-one meromorphic correspondence $g : M \to N$ where N is a complex submanifold of some $\mathbf{P}^k$. In particular, the above theorem applies to every projective algebraic manifold M.

It is easy to see that some restriction on M is required. To see this, consider the Hopf surface $M = \mathbf{C}^2 - \{0\}/\mathbf{Z}$ where an integer $n \in \mathbf{Z}$ acts on $\mathbf{C}^2 - \{0\}$ via $z \mapsto 2^n z$. The quotient M is compact, and the universal cover $\mathbf{C}^2 - \{0\} \xrightarrow{\pi} M$ is a domain in $\mathbf{C}^2$, yet this domain is not Stein. The theorem is motivated by the existence of algebraic varieties of the form D/Γ, where D is a Stein manifold and Γ is a properly discontinuous group of holomorphic automorphisms.

The proof of the above theorem is based on the following lemma.

LEMMA. *Let M be a compact Moishezon manifold. Suppose $D \xrightarrow{\pi} M$ is a covering of M by a connected complex manifold D. Then D is the "domain of existence" of a family $\mathfrak{F}$ of meromorphic functions on D, in the following sense: Suppose D' is a connected analytic space which contains D as an open subset. If each function in $\mathfrak{F}$ can be extended to a meromorphic function on D', then D' must coincide with D.*

Given this lemma, the theorem is a consequence of the work of E. E. Levi [7], Docquier and Grauert [1] and Kajiwara-Sakai [5]. Namely, suppose that D is a domain in a Stein manifold X (or, more generally, suppose that (D, φ, X) is a domain spread over a Stein manifold X). The above lemma immediately implies that (D, φ, X) is its own envelope of meromorphy over X with respect to the family $\mathfrak{F}$. Consequently, by lemma 5 of Kajiwara-Sakai [5], D is Stein. They prove, using the fundamental result of E. E. Levi [7] on meromorphic

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