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THE UNCERTAINTY PRINCIPLE IN RECONSTRUCTING FUNCTIONS FROM PROJECTIONS

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§1. Introduction.

Suppose $f(x, y)$ vanishes for $x^2 + y^2 > 1$ and belongs to L^2 over the unit disc D, $x^2 + y^2 \leq 1$. We rotate f through an angle θ by the change of variables,

$$(1) \qquad \begin{cases} x = t\cos\theta - u\sin\theta \\ y = t\sin\theta + u\cos\theta \end{cases} \qquad \begin{cases} t = x\cos\theta + y\sin\theta \\ u = -x\sin\theta + y\cos\theta \end{cases}$$

and then integrate on u to define the projection of f in the direction θ:

$$(2) \qquad P_f(t, \theta) = \int_{-\infty}^{\infty} f(t\cos\theta - u\sin\theta,\, t\sin\theta + u\cos\theta)\, du.$$

We note that

$$(3) \qquad P_f(t, \theta + \pi) = P_f(-t, \theta).$$

The integral in (2) may not exist for every t but a simple application of Schwarz's inequality shows that

$$(4) \qquad \int_{-1}^{1} \frac{|P_f(t, \theta)|^2\, dt}{\sqrt{1 - t^2}} \leq 2 \iint_D |f(x, y)|^2\, dx\, dy.$$

There is a well-known relation between the Fourier transform of the projection and the two-dimensional Fourier transform of f. We have from (2) and (1)

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