

THE UNCERTAINTY PRINCIPLE IN RECONSTRUCTING FUNCTIONS FROM PROJECTIONS

B. F. LOGAN

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§1. Introduction.

Suppose $f(x, y)$ vanishes for $x^2 + y^2 > 1$ and belongs to L^2 over the unit disc D , $x^2 + y^2 \leq 1$. We rotate f through an angle θ by the change of variables,

$$(1) \quad \begin{cases} x = t \cos \theta - u \sin \theta \\ y = t \sin \theta + u \cos \theta \end{cases} \quad \begin{cases} t = x \cos \theta + y \sin \theta \\ u = -x \sin \theta + y \cos \theta \end{cases}$$

and then integrate on u to define the projection of f in the direction θ :

$$(2) \quad P_f(t, \theta) = \int_{-\infty}^{\infty} f(t \cos \theta - u \sin \theta, t \sin \theta + u \cos \theta) du.$$

We note that

$$(3) \quad P_f(t, \theta + \pi) = P_f(-t, \theta).$$

The integral in (2) may not exist for every t but a simple application of Schwarz's inequality shows that

$$(4) \quad \int_{-1}^1 \frac{|P_f(t, \theta)|^2 dt}{\sqrt{1-t^2}} \leq 2 \iint_D |f(x, y)|^2 dx dy.$$

There is a well-known relation between the Fourier transform of the projection and the two-dimensional Fourier transform of f . We have from (2) and (1)

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