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CORRECTION

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To: Leon W. Green, *The gêneralized geodesic flow*, **41**(1974), 115–126

G. A. Margulis has pointed out that there exist compact manifolds of $\frac{1}{4}$-pinched negative curvature for which the generalized geodesic flow of frames in the principal bundle is not ergodic (I am indebted to D. V. Anosov for communicating this observation of Margulis to me.) Manifolds covered by complex hyperbolic spaces afford such examples. These manifolds do not have *strictly* $\frac{1}{4}$-pinched sectional curvatures. However, as Margulis and Anosov observe, it is not apparent in my article that strict pinching is used other than to prove smoothness of the horocycle fields, and the symmetric spaces certainly have smooth horocycle fields.

But the hypothesis that the sectional curvatures are strictly $\frac{1}{4}$-pinched is used in my proof at another point, which needs elaboration. Namely, the fact is tacitly used that the restricted homogeneous holonomy group of such a manifold is $SO(n)$. As far as I know, this result doesn't appear explicitly in the literature, but it may be traced through the Table II given by A. Gray [2]. The possibility that the group might be $U(n)$, not eliminated in this table, is covered by a theorem of M. Berger. He proved in [1] that, if a Kähler manifold, considered with its real Riemannian structure, has positive sectional curvatures δ-pinched, then $\delta \leq \frac{1}{4}$. His proof, which is purely algebraic, applies equally well to the case of negative curvature.

Thus the assertions throughout my proof of Theorem 3.6 to the effect that certain collections of vector fields generate the Lie algebra of $SO(n)$ turn out still to be true, but the arguments must be augmented by the above-cited facts about the holonomy group. For example, in the last paragraph of that proof, the fields $\{E_i^n\}$ generate the holonomy algebra, which in this case is $SO(n)$.

These comments lead naturally to the CONJECTURE: Let M be a compact, oriented Riemannian manifold with non-positive curvature. If B is a reduction of the principal bundle of orthonormal frames to a holonomy subbundle, the generalized geodesic flow in B is ergodic.

Clearly, more cannot be expected, since one cannot hope to approach more frames by the flow than one can with broken geodesics.

References

1. M. Berger, *Pincement riemannien et pincement holomorphe*, Annali Scuola Norm. Sup. Pisa **14**(1960), 151–159.

2. A. Gray, *Structures on Riemannian manifolds*, Differentialgeometrie im Grossen, Tagungsbericht des Math. For. Oberwolfach, 4, Mannheim 1971, 145–153.

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