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TRANSITIVE HOROCYCLES FOR FUCHSIAN GROUPS

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1. Introduction. A Fuchsian group G is a discrete group of bilinear transforms of the form

$$V(z) = \frac{az + b}{cz + d}, \qquad ad - bc = 1,$$

each of which preserve a disc (or half plane). We consider only groups which preserve the unit disc $\Delta = \{z : |z| < 1\}$ and none of whose transforms, except the identity, fix infinity (any Fuchsian group is conjugate to such a group). In this case the elements of G are of the form

$$V(z) = \frac{az + \bar{c}}{cz + \bar{a}}, \qquad |a|^2 - |c|^2 = 1, \quad c \neq 0. \tag{1}$$

The isometric circle of V is the circle $\{z : |cz + \bar{a}| = 1\}$ being the set of points z for which $|V'(z)| = 1$.

The set of points on $\partial\Delta$ at which G does not act discontinuously is the limit set $L(G)$. It is well known that $L(G)$ is the set of points of accumulation of centers of isometric circles belonging to transforms in G [4, p. 42]. The approximation is, however, not uniform and some interest has recently been shown in a class of limit points called points of approximation which are approximated very well by centers of isometric circles [3, 7]. In this paper we will consider a larger class of limit points for which the approximation is still good, derive some properties and use our results to investigate a transitivity question for Fuchsian groups. In section 3 we describe how our results may be extended to Kleinian groups.

We denote by D the Ford fundamental region for G, being the set of points in Δ exterior to all isometric circles of transforms in G. Note that D is also the Dirichlet fundamental polygon for G centered at the origin [6, p. 151]. We set $e = \partial D \cap \{|z| = 1\}$ and $E = \cup_{V \in G} V(e)$.

A horocycle is a euclidean circle which is internally tangent to $\partial\Delta$. The point of contact with $\partial\Delta$ is called the *point at infinity* on the given horocycle. The horocycle with euclidean radius r, $0 < r < 1$, and point at infinity ξ will be denoted $C(\xi, r)$.

Our first result is the following.

THEOREM 1. *Let G be a Fuchsian group and $\xi \in \partial\Delta$ then the following statements are equivalent.*

(i) $\xi \notin E$

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