Vol. 42, No. 2 DUKE MATHEMATICAL JOURNAL© June 1975

THE ACTION OF FREE G-BORDISM ON G-BORDISM

R. PAUL BEEM

§1. Introduction. In this note, we show, for finite abelian groups G, that G-bordism algebras are algebras over the Hopf algebra of free G-bordism. For example, if $N_*(G)$ and $\hat{N}_*(G)$ denote the G-bordism of closed manifolds with unrestricted isotropy subgroups and free actions respectively, then there is a module pairing

$$\Pi : \hat{N}_*(G) \otimes_{N_*} N_*(G) \to N_*(G)$$

and, if Δ denotes the coproduct of $\hat{N}_*(G)$, then the following diagram commutes:

$$\begin{tikzcd}
\hat{N}_*(G) \otimes (N_*(G) \otimes N_*(G)) \arrow[r, "1 \otimes \text{mult.}"] \arrow[d, "\Delta \otimes 1 \otimes 1"] & \hat{N}_*(G) \otimes N_*(G) \arrow[d, "\Pi"] \\
\hat{N}_*(G) \otimes \hat{N}_*(G) \otimes N_*(G) \otimes N_*(G) \arrow[d, "1 \otimes \text{twist} \otimes 1"] & N_*(G) \\
\hat{N}_*(G) \otimes N_*(G) \otimes \hat{N}_*(G) \otimes N_*(G) \arrow[r, "\Pi \otimes \Pi"] & N_*(G) \otimes N_*(G). \arrow[u, "\text{mult.}"]
\end{tikzcd}$$

(All tensor products will be over N_*, the unoriented bordism ring.)

In §2, we define the pairing Π for slightly more general G-bordism modules and show that most of the N_*-structures and homomorphisms of [8] can be extended over $\hat{N}_*(G)$. In §3 we show that these modules are, in fact, algebras and that $\hat{N}_*(G)$ acts on the multiplicative structure via its coproduct. In §4 we look at the example $G = Z_2$, compute some structure and, as an application, characterize the image of $N_*(S^1)$ in $N_*(Z_2)$.

The general reference for the first two sections is [8] and for the last section, [7].

The author wishes to express his thanks to R. E. Stong for mentioning this problem and for his many helpful suggestions. In particular, the proof presented here of Theorem 3 is his improved version of an earlier effort of the author.

§2. Preliminaries on G-bordism. Let G be a finite abelian group. $N_*(G)(\mathfrak{F}, \mathfrak{F}')$ will denote the N_*-module of bordism classes of compact smooth manifolds (with boundary) with G action with isotropy subgroups in the pair of families $(\mathfrak{F}, \mathfrak{F}')$ of subgroups of G.

Specifically, the definitions are:

1. A collection $\mathfrak{F}$ of subgroups of G is a "family" if whenever H is in $\mathfrak{F}$ and K is a subgroup of H, then K is in $\mathfrak{F}$.

Received October 25, 1974.