F-PROJECTIVE HOMOTOPY AND F-PROJECTIVE STABLE STEMS

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Let F denote either the classical real (R), complex (C) or quaternionic (Q) numbers. Let

$$h_F : S^{d_F(n+1)-1} \to FP^n$$

denote the Hopf fiber bundle projection with fiber S^{d_F-1} for $n \geq 0$ where d_F is the real vector space dimension of F. We work in the topological category of pointed spaces and pointed maps. Given a space X and a positive integer m, we define

$$\pi_m{}^F(X) \begin{array}{ll} = h_F{}^{\#}[FP^n, X] & \text{if } d_F(n+1) = m+1 \\ = 0 & \text{if } d_F \text{ does not divide } m+1 \end{array}$$

The sets $\pi_m{}^F(X)$ form the F-projective homotopy of X. If X is either $[m + 1/2]$-connected or an associative H-space with an inverse, then $\pi_m{}^F(X)$ is a subgroup of $\pi_m(X)$. Note that $\pi_m{}^Q(X) \subseteq \pi_m{}^C(X) \subseteq \pi_m{}^R(X)$ via the maps $\bar{h}_C : RP^{2s+1} \to CP^s$ and $\bar{h}_Q : CP^{2t+1} \to QP^t$ induced from h_C and h_Q.

For example, let us take X to be a topological group G. Then $\pi_{d_Fn-1}{}^F(G)$ can be regarded as the set of equivalence classes of principal G-bundles ξ over S^{d_Fn} with $\theta^*\xi$ trivial where $\theta: FP^n \to S^{d_Fn}$ collapses FP^{n-1}. Also, any odd-dimensional homotopy class which can be halved must be R-projective since $2\pi_m(X) \subseteq \pi_m{}^R(X)$ for m odd. (See [6] for an application).

In this paper we determine F-projective stable stems in dimensions ≤ 6. The R-projective stable stems have been found by Rees [7] and Strutt [8] through dimension 5 and through higher dimensions by Bredon [1] using various methods. We also describe the F-projective homotopy of the real and complex projective spaces in §3 in terms of the F-projective homotopy of spheres. We thank a referee for his helpful suggestions.

§1. Stable Stems and Cohomology Operations. We first establish some notation. We use Z_m for Z/mZ. The coefficient group for cohomology is understood to be Z_2 whenever omitted. We let α, β, and γ denote generators for $H^*(RP^\infty)$, $H^*(CP^\infty; Z)$; and $H^*(QP^\infty; Z)$ respectively. Let $\rho_t : H^*(\ ; Z) \to H^*(\ ; Z_t)$ be the morphism corresponding to reduction $Z \to Z_t$. $P_p{}^i$ represents the ith mod p Steenrod reduced power. An element $\rho \in \pi_t{}^s$ of the stable

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