A GENERAL CALCULUS OF PSEUDODIFFERENTIAL OPERATORS

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Contents:

Introduction. Pseudodifferential operators have become an indispensable tool in the study of linear partial differential equations. In principle, the larger the class of pseudodifferential operators for which one has a usable operational calculus, the wider the range of applications should be. In this paper we develop such a calculus for very general classes of operators, and include a few applications for illustration. We begin with a very brief survey of some of the existing theory and applications; the articles of Calderón [11] and Nirenberg [39] contain much additional information.

Calderón and Zygmund [14] applied a calculus of singular integral operators to the study of elliptic equations. Calderón [9], [10] then used this calculus to obtain the first very general theorems on uniqueness in the Cauchy problem and existence of solutions for systems with variable coefficients.

The calculus of singular integral operators was refined and recast into a theory of pseudodifferential operators by Kohn and Nirenberg [32], Seeley [45], Unterberger and Bokobza [53] and Hörmander [25]. In this form the theory deals with operators whose symbols $a(x, \xi)$ have asymptotic expansions as sums of symbols which are positive homogeneous in ξ of decreasing orders; it includes parametrices for elliptic operators. This theory has been used to establish

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