ON A CLASS OF HYPERSURFACES OF $\mathbf{R}^{2n+1}$

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Introduction. In this paper we give a partial answer to the question regarding which complete Kählerian manifold can be immersed isometrically into Euclidean space as a real hypersurface.

As is well known, there is a broad class of Riemann surfaces which satisfy the above criterion; therefore, any Kählerian manifolds which are Kählerian products of any such Riemann surfaces and complex Euclidean space satisfy the same criterion in the natural fashion. What follows states a kind of converse.

THEOREM. *Let M^{2n} be a complete connected Kählerian manifold of real dimension $2n$, and furthermore, let M^{2n} be immersed by f isometrically into $\mathbf{R}^{2n+1}$. Then M^{2n} is a product of a Riemann surface and $\mathbf{C}^{n-1} = \mathbf{R}^{2(n-1)}$, provided either*

Case 1. M^{2n} has non-negative scalar curvature, or

Case 2. M^{2n} has strictly negative scalar curvature, or

Case 3. The immersion f is real analytic.

Furthermore, the immersion f splits into an immersion of the Riemann surface into $\mathbf{R}^3$ and the natural identification of $\mathbf{C}^{n-1} = \mathbf{R}^{2(n-1)}$ with the orthogonal complement of $\mathbf{R}^3$ in $\mathbf{R}^{2n+1}$. Thus the study of such Kählerian hypersurfaces is reduced to that of the usual surfaces in $\mathbf{R}^3$ under each condition given in the theorem. Now as a special case, we have

COROLLARY. *Let S^2 be the standard sphere of dimension 2, and let $M^{2(n-1)}$ be any complete connected Kählerian manifold of dimension $2(n-1)$. Then $S^2 \times M^{2(n-1)}$ can be immersed isometrically into $\mathbf{R}^{2n+1}$ if and only if $M^{2(n-1)} = \mathbf{R}^{2(n-1)} = \mathbf{C}^{n-1}$. The immersion is the product of the standard imbeddings of S^2 into $\mathbf{R}^3$ and $\mathbf{C}^{n-1}$ onto $\mathbf{R}^{2(n-1)}$.*

Kählerian manifolds which are isometrically immersed in Euclidean space as hypersurfaces are called real Kählerian hypersurfaces. Takahashi [8] and Ryan [7] have recently studied such Kählerian manifolds. In particular, Ryan [7] has determined all the complete connected real Kählerian hypersurfaces of spaceforms of non-zero curvature. Furthermore, he has obtained the result that any complete real Kählerian hypersurface of Euclidean space is locally either flat or a product of an open subset of a flat space and an open subset of a Riemann surface. Our result is a further pursuit of this line based on a lemma of Takahashi [8].

Preliminaries. Let M^{2n} be a connected complete Kählerian manifold occurring as a hypersurface in $\mathbf{R}^{2n+1}$. We denote by $\tilde{\nabla}$ and ∇ the Riemannian

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