ZEROS OF FUNCTIONS IN THE BERGMAN SPACES

CHARLES HOROWITZ

1. Introduction. A function $f(z)$ analytic in the unit disc $|z| < 1$ is said to belong to the Hardy space H^p, $0 < p < \infty$, if

$$\mathfrak{M}_p(f, r) = \left\{\frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p \, d\theta\right\}^{1/p}$$

remains bounded as $r \to 1$. The space of bounded analytic functions is called H^∞. It has been known for a long time that the zeros $\{z_k\}$ of nontrivial H^p functions are completely characterized by the Blaschke condition

$$\sum_{k=1}^{\infty} (1 - |z_k|) < \infty .$$

In particular, all of the H^p spaces admit the same zero sets. (See, e.g., [1].)

It is a problem of long standing to describe the zero sets of analytic functions f for which $|f(z)|^p$ is integrable with respect to the area measure $dA = 1/\pi \, dx \, dy$ over the disc. Such a function is said to belong to the Bergman space A^p, and we define its A^p norm by the equation

$$||f||_p^p = \int_{|z|<1} |f(z)|^p \, dA .$$

It is clear that $H^p \subset A^p$, and that $f \in A^p$ if and only if

$$\int_0^1 \{\mathfrak{M}_p(f, r)\}^p \, dr < \infty .$$

For convenience, we define $A^\infty = H^\infty$. It should be noted that $A^q \subset A^p$ if $p < q$. One can show without difficulty that A^p is a norm-closed subspace of the L^p space constructed over the disc with respect to area measure. In particular, A^p is a Banach space for $1 \leq p \leq \infty$.

In this paper, we obtain some results on the structure of the zero sets of A^p functions. We show that these "A^p zero sets" are quite different from the Blaschke sequences, or H^p zero sets. Roughly, our main theorems are as follows.

THEOREM 1. *If $0 < p < q \leq \infty$, then there is an A^p zero set which is not an A^q zero set.*

THEOREM 2. *If $0 < p < \infty$, then the union of two A^p zero sets is not in general an A^p zero set.*

Received March 1, 1974. This paper represents part of the author's doctoral thesis written at the University of Michigan under the direction of Professor Peter L. Duren, whose help is greatly appreciated.