OPERATORS WITH SPECTRUM IN A C^1-JORDAN CURVE

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An important part of the theory of spectral operators is devoted to finding conditions sufficient to insure spectrality. There has been a considerable amount of work done in this direction, particularly when $\sigma(T)$ lies in a C^2-Jordan curve. In this paper we are concerned with an operator on a Hilbert space H whose spectrum is in a C^1-Jordan curve. Dunford [4] has shown that if a spectral operator T with spectrum in a C^2-Jordan curve satisfies a growth condition then T is of finite type. Stampfli [10] has given an example to show that the above result is not necessarily true when $\sigma(T)$ is in a rectifiable Jordan curve. We show that Dunford's result is valid when $\sigma(T)$ is in a C^1-Jordan curve, thus closing the gap. This result is used to find sufficient conditions for an operator with $\sigma(T)$ in C^1 curve to be spectral. Our result is an analogue of a result of Dunford [4] for C^2-Jordan curve. Apart from the technical difficulties which arise because of our consideration of C^1-Jordan curves, the techniques are quite similar to those used in Bartle [1], Dunford [2, 3, 4], Schwartz [7], Stampfli [8, 9, 10], and Yoshino [11].

The Banach algebra of all bounded linear operators on a Hilbert space H is denoted by $B(H)$. The spectrum, the continuous spectrum and the resolvent set of an operator $T \in B(H)$ is denoted by $\sigma(T)$, $\sigma_c(T)$ and $\rho(T)$ respectively.

1. Preliminaries and $\mathbf{C^1}$ Jordan Curves. We begin by introducing a generalization of the spectrum and the resolvent set due to Dunford [2, 3]. Let $x \in H$, then $R(z; T)x = (zI - T)^{-1}x$ is an analytic vector valued function for $z \in \rho(T)$. A vector valued function $f(z)$ is an analytic extension of $R(z; T)x$ if it is defined and is analytic on an open set $D(f)$ containing $\rho(T)$ and if $(zI - T)f(z) = x$ for all $z \in D(f)$. If $R(z; T)x$ has single valued extension property (i.e., each pair of its analytic extensions coincides on their common domain) then we define a maximal single valued extension of $R(z; T)x$ by taking the union of all analytic extensions of $R(z; T)x$ and we designate it by $\hat{x}(z)$. Now we set $\rho(T, x) = \{z : \hat{x}(z)$ is analytic at $z\}$ and $\sigma(T, x) =$ complement of $\rho(T, x)$. The operator T has the single valued extension property if $R(z; T)x$ has this property for all $x \in H$. The reader is referred to [5] for a discussion of the properties of $\sigma(T, x)$.

Throughout this paper Γ will denote a C^1 Jordan curve of length 2π and $g : S^1 = \{z : |z| = 1\} \to \Gamma$ will stand for arc length parameterization of Γ. Let $n(s)$ denote the unit normal (perpendicular to the tangent and directed to the

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