GENERALIZED REFLECTION PRINCIPLES FOR PARABOLIC EQUATIONS IN ONE SPACE VARIABLE

DAVID COLTON

I. Introduction. In [1] the author has obtained reflection principles for solutions to the parabolic equation

$$u_{xx} + a(x, t)u_x + b(x, t)u = u_t \tag{1.1}$$

satisfying the boundary condition $u(0, t) = 0$, and for solutions of

$$u_{xx} + q(x, t)u = u_t \tag{1.2}$$

satisfying $u_x(0, t) = 0$, under the assumption that the coefficients in (1.1) and (1.2) are analytic. It is the purpose of this paper to extend these results to include the case when $u(x, t)$ is a solution of (1.1) satisfying the boundary data

$$\alpha(t)u(0, t) + \beta(t)u_x(0, t) = f(t) \tag{1.3}$$

where $\alpha(t)$, $\beta(t)$, and $f(t)$ are analytic functions and $\beta(t) \neq 0$ for $0 < t < t_0$, t_0 being a positive constant. This result provides an analogue for parabolic equations of Lewy's reflection principle for elliptic equations ([5]). Our approach to the reflection problem (1.1), (1.3), is based on the construction of an integral operator which maps solutions of the heat equation onto solutions of parabolic equations with variable coefficients. However it is of interest to note that our results are new even for the heat equation. We also want to emphasize that the reflection principle obtained here is valid for strong (in particular not necessarily analytic) solutions of (1.1). In this context we observe that for analytic solutions of parabolic equations in one space variable with analytic coefficients reflection principles are trivial in the sense that any analytic solution of a parabolic equation in one space variable can be analytically continued into a strip bounded by the characteristics, regardless of what the (analytic) data is on the t axis ([2], [4]).

II. Reduction to Canonical Form. We consider (1.1) under the assumption that the coefficients $a(x, t)$ and $b(x, t)$ are analytic in the rectangle $D = \{(x, t): -x_0 < x < x_0, 0 < t < t_0\}$ where x_0 and t_0 are positive constants. Setting

$$u(x, t) = v(x, t) \exp\left\{-\frac{1}{2}\int_0^x a(s, t)\, ds\right\} \tag{2.1}$$

Received October 26, 1973.

This research was supported in part by AFOSR Grant 71–2205A and AFOSR Grant 74-2592.