STABLE EXISTENCE OF FINITE GROUP ACTIONS ON MANIFOLDS

ALLAN L. EDMONDS

Let G denote the finite cyclic group of prime order p. The purpose of this note is to prove the following result (all homology groups are taken with integer coefficients):

THEOREM. *Let M be a closed, simply-connected, smooth n-manifold with Euler characteristic zero, such that $H_*(M)$ contains no p-torsion. Then there is a free smooth action of G on the stable normal disk bundle of M, of fiber dimension at least $n + 1$.*

The proof is accomplished via a sequence of three reductions. At the end we also indicate examples which show that no one of the hypotheses may be omitted and that M itself need not support such an action.

1. First Reduction. In this section M denotes a closed, simply-connected, smooth n-manifold and G denotes an arbitrary finite group. Let V be the stable normal disk bundle of M of dimension $m \geq 2n + 1$, $m \geq 6$, which we identify as a tubular neighborhood of M in euclidean m-space.

PROPOSITION 1. *If there is a finite CW n-complex X of the homotopy type of M on which G acts freely and cellularly, then there is a free smooth action of G on V.*

Proof. Note that V is a simply-connected m-manifold with simply-connected boundary and trivial tangent bundle. It is easy to construct a handle-body W which is a simply-connected m-manifold with simply-connected boundary and trivial tangent bundle, a free action of G on W, and an equivariant homotopy equivalence $W \to X$. The proof is then completed by the standard observation that V is diffeomorphic to W, using general position, immersion classification, and the h-cobordism theorem.

COROLLARY. *Let G be a finite group with periodic cohomology. Then G acts freely on $S^n \times D^{n+1}$ for appropriate n.*

Proof. According to Swan [6, Theorem A] there is a finite CW complex of the homotopy type of S^n on which G acts freely and cellularly. By Proposition 1, G acts freely on the stable normal disk bundle of S^n, namely, $S^n \times D^{n+1}$.

2. Second Reduction. In this section let G be any finite group, with integral group ring $\Lambda = \mathbf{Z}[G]$, and let X be a finite, simply-connected, CW n-complex, with cellular chain complex $C_* = C_*(X)$.

Supported in part by National Science Foundation grant GP36418X1.
Received September 28, 1973. Revisions received December 10, 1973.