NONSTANDARD HULLS OF THE CLASSICAL BANACH SPACES

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In this paper we consider the structure of the nonstandard hulls of the classical Banach spaces (e.g., L^p spaces, $C(X)$ spaces) using the methods developed in [4] and [2].

For convenience we adopt the notational conventions and the framework for nonstandard analysis described in [2]. In particular throughout this paper $^*\mathfrak{M}$ will denote an enlargement of a set-theoretical structure $\mathfrak{M}$ and we assume that $^*\mathfrak{M}$ is at least $\aleph_1$-saturated.

If (S, Σ, μ) is a measure space (with μ a positive measure), we let $L^p(\mu)$ denote the corresponding Banach lattice of p'th power integrable, real-valued functions $(1 \leq p < \infty)$ and $L^\infty(\mu)$ denote the Banach lattice of μ-essentially bounded, measurable real-valued functions. The analogous Banach spaces of complex-valued functions will be denoted by $L^p{}_{\mathbf{C}}(\mu)$. If S has n elements and μ is the equal-weight measure, we denote $L^p(\mu)$ and $L^p{}_{\mathbf{C}}(\mu)$ by $l^p(n)$ and $l^p{}_{\mathbf{C}}(n)$ respectively. In general, when the measure space (S, Σ, μ) is completely atomic, we denote the corresponding L^p-spaces by $l^p(S)$ or $l^p{}_{\mathbf{C}}(S)$. When S is the set $N = \{1, 2, \cdots\}$, we simply write l^p and $l^p{}_{\mathbf{C}}$. We adopt similar conventions for the spaces of convergent sequences $c_0(S)$ and c_0 and for the corresponding complex spaces.

If X is a compact Hausdorff space, we denote by $C(X)$ (respectively $C_{\mathbf{C}}(X)$) the space of continuous, real-valued (respectively complex-valued) functions on X, with the supremum norm. Recall that $C(X)$ is a Banach lattice when given the pointwise ordering.

Section 1. Abstract L^p-spaces; M-spaces.

Definition 1.1. Let (L, ρ) be a Banach lattice.

(i) For $1 \leq p < \infty$, (L, ρ) is said to be an *abstract L^p-space* if $0 \leq u, v \in L$ with $\inf(u, v) = 0$ implies $\rho(u + v)^p = \rho(u)^p + \rho(v)^p$.

(ii) (L, ρ) is said to be an *abstract M-space* if $0 \leq u, v \in L$ with $\inf(u, v) = 0$ implies $\rho(u + v) = \max(\rho(u), \rho(v))$.

It is known that if (L, ρ) is an abstract L^p-space, then for some measure space (X, Σ) and positive measure η, (L, ρ) is Riesz isometric to $L^p(X, \Sigma, \eta)$. (For $p = 1$ this is Kakutani's characterization of abstract L-spaces. This can be found in [3]. For $1 < p < \infty$, this follows from the results of Marti [10] by noting, as Bernau [1] and Meyer–Nieberg [11] have done, that Marti's condition of weak sequential completeness is unnecessary.) If (L, ρ) is an

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