CURVATURE STRUCTURES IN COMPLEX MANIFOLDS

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This paper deals mainly with the study of complex manifolds via curvature structures. Such a study in completely generalised form has been done for Riemannian manifolds by Kulkarni [7], [8], where he has shown how mostly curvature preserving maps reduce to conformal maps. We ask similar questions for manifolds with an additional structure and find partial answers.

Our results are derived by using the concept of holomorphic bisectional curvatures introduced by Goldberg and Kobayashi [2] for Kaehler manifolds and extended later for nearly Kaehler manifolds by A. Gray [3].

To begin with, we describe in brief the basic concepts.

1. Let (M, h, J) be the n-dimensional (complex) C^∞-hermitian manifold to be denoted in short by M, and let $\langle , \rangle$ denote the inner product defined by h. Also let $G_2(M)$ be the Grassman bundle of 2-planes and let $\bar{G}_2(M)$ be the collection of J-invariant planes; it is known that $\bar{G}_2(M)$ is a holomorphic bundle over M whose fibers are complex projective spaces of dimension $n - 1$.

M is called a Kachlerian manifold if the Kaehler form defined by $\phi(X, Y) = h(JX, Y)$, where $X, Y \in \mathfrak{X}(M)$, is closed.

We also define a function $\tilde{H}_B : \bar{G}_2(M) \times \bar{G}_2(M) \to R$ by setting

$$\tilde{H}_B(\bar{\sigma}, \bar{\sigma}') = \frac{\langle B(X, JX)Y, JY\rangle}{\langle X, X\rangle\langle JY, JY\rangle}$$

which gives the holomorphic bisectional curvature at points of M [3].

We list the following remarks resulting from our formulations

(1) $\tilde{H}_B$ restricted to the diagonal of $\bar{G}_2(M) \times \bar{G}_2(M)$ coincides with H_B .

(2) If $\tilde{H}_B$ is a constant function, then H_B is also constant.

(3) On a Kaehler manifold M with Riemannian curvature structure R, for every pair $\bar{\sigma} = \{X, JX\}$, $\bar{\sigma}' = \{Y, JY\}$, $\tilde{H}_R(\bar{\sigma}, \bar{\sigma}') = K_R(\sigma) + K_R(\sigma')$ where $\sigma = \{X, Y\}$ and $\sigma' = \{X, JY\} \in G_2(M)$.

It is easy to check that on any (M, h, J) besides having the Riemannian curvature structure, one can also have the following curvature structures [7].

(1) $I(X, Y)Z = \langle X, Z\rangle Y - \langle Y, Z\rangle X$,

(2) $\mathcal{R}\mathrm{ic}\,(X, Y)Z = \{\mathrm{Ric}\,(X, Z)Y - \mathrm{Ric}\,(Y, Z)X + \langle X, Z\rangle \mathrm{Ric}.\, Y - \langle Y, Z\rangle \mathrm{Ric}.\, X\}$,

(3) $C = R - \dfrac{1}{n-2}\mathcal{R}\mathrm{ic} + \dfrac{Sc}{(n-1)(n-2)} I$,

where Ric (X, Y) is the Ricci tensor with respect to the Riemannian tensor R, Ric. is the corresponding linear transformation and C is the conformal tensor.

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