THE GENERALIZED GEODESIC FLOW

LEON W. GREEN

Introduction. By a generalized geodesic flow I mean a horizontal lift of the geodesic flow on a complete Riemannian manifold into the bundle of orthonormal frames. In the case of constant curvature, these are particular examples of flows in frame bundles considered by Arnol'd, and they also fall under the general analysis for flows on homogeneous spaces carried out by C. C. Moore (See [2] and [12].). My object is to investigate the case of such flows on not necessarily symmetric spaces, and, in doing so, to extend various techniques of a Lie algebraic character to general Riemannian manifolds.

The main results are the construction of horocycle flows and the resulting Bruhat-like decomposition of the frame bundle (Section 2) and the proof (Section 3) of the following theorem.

THEOREM. *Let M be an oriented compact Riemannian manifold with $\frac{1}{4}$-pinched negative curvature. Then any generalized geodesic flow in its bundle of orthonormal frames is weakly mixing.*

It then follows from a theorem of K. Thomas [14] and the known Anosov character of the geodesic flow on the unit tangent bundle that each map of the frame flow is actually a Kolmogorov automorphism. The generalized geodesic flows are restrictions of Anosov actions to one parameter subgroups, but despite the ergodicity of Anosov actions [13], specific ergodic one parameter subgroups cannot usually be singled out. (After the present article was prepared, D. V. Anosov informed me that M. Brin and Ja. Pesin have proved the ergodicity of a generalized geodesic flow for metrics which are perturbations of those with constant negative curvature. This result is announced in Uspehi Mat. Nauk 1973, N. 3, 4 and details are to appear in Isvestia Akad. Nauk.)

An outline of these results was given at the Conference in Topological Dynamics in honor of Professor Gustav A. Hedlund, Yale, June 21, 1972. The interested reader will find a discussion of the group theoretical motivation of these results, together with a more extensive bibliography, in the proceedings of that conference [8]. A very preliminary version was announced in [7].

1. Structure equations and horospheres. Let $\mathbf{F}M$ (abbreviated $\mathbf{F}$) be the bundle of orthonormal frames of the complete, n-dimensional, connected, C^∞ Riemannian manifold M. $\mathbf{F}$ is a principal $\mathrm{O}(n, \mathbf{R})$-bundle over M, and each $f \in \mathbf{F}$ may be interpreted as an isomorphism of $\mathbf{R}^n$ onto $T_{\pi(f)}M$, the

Received September 10, 1973. The author was supported in part by National Science Foundation grants GP-27670 and GP-29321 A2.