ON CONSTRUCTING MONOTONE AND UV^1 MAPPINGS OF ARBITRARY DEGREE

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1. Introduction. This paper was stimulated by a question of Lacher and McMillan [5]. In particular, is there an inessential UV^1 map of S^m onto itself? If $m \leq 4$, then the answer is no [5]. Theorem 2 below shows that if $m \geq 5$, then there is a UV^1 map of S^m onto itself of any degree. Since a UV^0 map is monotone, another application of Theorem 2 is that if $m \geq 3$, then there is a monotone map of S^m onto S^m of arbitrary degree. This gives a counterexample to [6; Theorem 4.5], which states that every monotone map between closed 3-manifolds can be approximated by a simplicial monotone mapping. For if Schoenefeld's result were true, then since every simplicial monotone mapping between closed 3-manifolds has degree -1, 0, or 1, every monotone mapping between closed 3-manifolds would have degree -1, 0, or 1, which contradicts Theorem 2 below. Theorem 5 shows that Soloway's criterion for compactness is the best possible for UV^1 mappings. Thus, Theorem 5 answers part of [4; Question III].

If K is a finite complex, then let $B_n(K)$ denote the n-th barycentric subdivision of K. If G is a collection of subsets of a set X, then let G^* denote the union of the members of G. If M^m is an m-manifold, then let Bd M^m denote the boundary of M^m.

DEFINITION. Let K and L be finite polyhedra. A mapping f from K into L will be called *almost simplicial* if for every $m \geq 0$ there is an integer n such that if $\sigma \in B_n(K)$, then $f(\sigma)$ is a subset of some member of $B_m(L)$.

Note that simplicial mappings are almost simplicial. Let B^m and S^m denote the m-ball and m-sphere respectively.

The main theorem of the paper and the central device used in proving Theorem 2 is the following theorem.

THEOREM 1. *Let M^m be a compact connected triangulated m-manifold. Let h be an almost simplicial map from* Bd M^m *into* Bd B^q. *Let $k = -1, 0,$ or 1. If $m \geq 2k + 3$ and M^m is k-connected, then there exists an almost simplicial UV^k map H from M^m onto B^q such that H extends h.*

Recall that a space X is called *k-connected* if $\pi_i(X) = 0$ for $i = 0, \cdots, k$. The definition of UV^k mapping is given in [3]. The techniques used in the proof of Theorem 1 are very similar to those used in [8]. Note that there is no restriction on the integer q in Theorem 1 so that the map H may raise dimension.

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