CONCERNING FIRST COUNTABLE SPACES, II

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In [15] the author gave necessary and sufficient conditions for the existence of dense developable and dense metrizable subspaces in first countable spaces. In addition, each Nagata space was shown to have a dense metrizable subspace and each semi-metric space was shown to have a dense developable subspace. In this paper the author continues the investigation begun in [15]. An example is given of a first countable T_2-space which has no dense developable subspace. Furthermore, it is shown that (1) each space with a base of countable order has a dense quasi-developable subspace and (2) each space with a base of countable order and which has the Baire property has a dense metrizable G_δ-subset. It follows that (i) for each subset M of the regular $w\Delta$-space (M-space) S with a G_δ-diagonal in which closed sets are G_δ's there exists a dense Moore (metrizable) subspace K of S such that $M \cap K$ is dense in M and (ii) each regular countably compact space with a G_δ-diagonal has a dense metrizable subspace.

By a development for a space S (all spaces are to be T_1) is meant a sequence $G_1, G_2, \cdots$ of open coverings of S such that for each point p and each open set D containing p there exists a positive integer n such that each element of G_n containing p is contained in D. A regular space having a development is a Moore space.

THEOREM 1 [15]. *The first countable space S has a dense developable subspace if and only if there exists a dense subspace X of S such that X is the union of countably many subsets X_i, where for each i no point of X is a limit point of X_i.*

THEOREM 2. *A hereditarily Lindelöf, nonseparable space has no dense developable subspace.*

Proof. Suppose Y is a dense developable subspace of the hereditarily Lindelöf space S. By Theorem 1, there exists a dense subspace X of Y such that $X = \cup X_i$, where for each i no point of X is a limit point of X_i. But, since S is hereditarily Lindelöf, it follows that X is countable. Hence, S is separable. Thus each hereditarily Lindelöf, nonseparable space has no dense developable subspace.

COROLLARY 2.1. *If Souslin spaces exist, they have no dense developable subspaces.*

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