PATH LIFTING FOR DISCRETE OPEN MAPPINGS

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1. Introduction. The path lifting problem for light open mappings has been considered by Stoïlow [5; 354], [6; 109], Whyburn [8; 186], Floyd [1; 574], and Martio, Rickman, and Väisälä [2; 2.7], [3; 3.12]. In this paper we study the path lifting for discrete open mappings in Euclidean n-space. The main result, Theorem 1, gives globally liftings which form a maximal set in two respects. Firstly, the number of liftings starting at given points is maximal under the condition that the number is locally bounded by the local index, and secondly, the liftings are maximal liftings. Theorem 1 has been applied by Väisälä [7] to the theory of quasiregular mappings. As a consequence we obtain for normal domains a finite number of liftings which cover the preimage of the given path. This type of result has been proved by Poleckii [4; Lemma 4] under a restrictive assumption. It turns out that his idea can partially be used also in the general case.

The proof of Theorem 1, which is constructive and by induction on the local index, also gives a new proof of the existence of path lifting for discrete open mappings in n-space.

2. Terminology and notation. Throughout the paper we assume that $f : G \rightarrow R^n$ is a continuous, discrete, and open mapping of a domain G in the n-dimensional Euclidean space R^n. A domain $D \subset G$ is a *normal domain* of f if $\bar{D}$ is a compact subset of G and $\partial fD = f\partial D$. A *normal neighborhood* of a point $x \in G$ is a normal domain $D \ni x$ of f such that $D \cap f^{-1}(f(x)) = \{x\}$. By [2; 2.10] every point in G has arbitrarily small normal neighborhoods.

For $x \in G$ we denote by $i(x, f)$ the *local index* of f at x [2; 6]. The *branch set* of f, i.e., the set of points in G where f fails to be a local homeomorphism, is denoted by B_f. We have $|i(x, f)| \geq 2$ if and only if $x \in B_f$ [2; 2.12]. From $\dim B_f = \dim fB_f \leq n - 2$ it follows that $G' \backslash B_f$ is a domain for any domain $G' \subset G$ and that either $i(x, f)$ is positive for all $x \in G$ or $i(x, f)$ is negative for all $x \in G$ [2]. In the former case f is sense-preserving and in the latter case sense-reversing. If D is a normal domain of f, we have for $y \in fD$

$$\sum_{x \in D \cap f^{-1}(y)} |i(x, f)| = N(f, D),$$

where $N(f, D) = \sup \{\text{card } D \cap f^{-1}(z) \mid z \in R^n\}$ [2]. Hence, especially if D is a normal neighborhood of x, we have for $y \in fD$

$$\sum_{u \in D \cap f^{-1}(y)} |i(u, f)| = |i(x, f)|.$$

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