ON ALMOST PERIODIC AND ALMOST AUTOMORPHIC DIFFERENCES OF FUNCTIONS

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A classical theorem by Bohr states that if f is a continuous almost periodic function of one real variable with values in the complex numbers, then $F(x) = \int_0^x f(t)\, dt$ is almost periodic if and only if it is bounded [3]. In the differentiable version the theorem states that a bounded continuously differentiable function is almost periodic if and only if the derivative is almost periodic.

It is well known [10] that the above mentioned theorem may be derived as a consequence of a theorem on differences which states that a bounded function F is almost periodic when all differences $F(a + x) - F(x)$ are almost periodic functions in the variable x. Indeed, the formula

$$F(a + x) - F(x) = \int_0^a f(t + x)\, dt$$

yields the almost periodicity of the differences when the almost periodicity of f is known.

It is a consequence of the above observations that theorems on differences offer the opportunity to generalize Bohr's theorem to the case where differentiable structures are not available or suitable. In this regard the valuable generalizations of Günzler should be noted [7]. However, our versions take on an entirely different direction altogether. Other recent authors with difference theorems relevant to this paper are Berg [1], Bochner [2], Doss [5], and Loomis [10]. We begin the exposition after the introduction by combining our ideas with an argument of Bochner and a theorem by Loomis.

This paper is intended to be a sequel to [12], where we extracted a number of facts about almost automorphic functions by topological methods.

1. Introduction. Let G be a topological group. Let $C(G)$ denote the continuous bounded functions on G with values in the field of complex numbers. Let f be any bounded function from G into the complex numbers. If α in G is a net such that $T_\alpha f(x)$, defined by the limit $\lim f(\alpha_i x)$, exists for every $x \in G$, then we shall say that α is *regular* for f. Recall that f is *almost automorphic* (a.a.) if $T_{\alpha^{-1}} T_\alpha f = f$ whenever α is a net regular for f and α^{-1} is a net regular for $T_\alpha f$. If for every such α in the defining condition the limits are necessarily uniform, then f is said to be *almost periodic* (a.p.) If for every net α regular for f there exists a net β regular for $T_\alpha f$ such that $T_\beta T_\alpha f = f$, then we say that f is *minimal*

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