FK-SPACES CONTAINING c_0

G. BENNETT AND N. J. KALTON

1. Introduction. We denote by ω the vector space of all sequences of complex numbers tolopogized by means of coordinatewise convergence. An *FK-space* E is a vector subspace of ω endowed with a complete metrizable locally convex topology under which the natural inclusion map $E \rightarrow \omega$ is continuous. FK-spaces have proved to be a useful tool in function theory [78] and, more recently, in the study of bases in Banach spaces [64]. Their most important application, though, to summability theory stems from Zeller's observation [76] that the convergence domain c_A, defined below, of a matrix A is an FK-space.

The bounded convergence domain of A, $m \cap c_A$, is also an FK-space but has received much less attention from the functional analytic viewpoint. One reason for this is that $m \cap c_A$, as an FK-space, is in general nonseparable and has a rather intractable dual space [80; 53]. To circumvent these difficulties Alexiewicz and Orlicz [5] used the theory of two-norm spaces; their path also has its shortcomings, however, in that the Hahn–Banach theorem is generally no longer valid in the two-norm setting. Our approach is to identify the two-norm topology as a certain Mackey topology which enables us to use the powerful duality theory of locally convex spaces. Furthermore, this method is applicable to FK-spaces in general and leads to results that should be of interest outside summability theory.

We proceed by introducing notation in Section 2 and listing some of the fundamental results from the theory of locally convex spaces. A new proof of the Orlicz–Pettis theorem on unconditional convergence leads to a slightly stronger result (Theorem 1) which is needed later. We close Section 2 with a brief bibliographical discussion of the theory of mixed topologies and their role in summability theory.

Section 3 brings a detailed account of FK-spaces containing c_0. The main results are the identification of a certain two-norm topology mentioned above (Theorem 5), a completeness theorem for the space l^1 (Theorem 3) from which the celebrated bounded consistency theorem follows, and a refinement of a result of Snyder on conull FK-spaces (Theorem 2). This section also contains several other results that should be of interest in themselves.

Section 4 deals with direct applications of the above theorems, and the main result (Theorem 7) yields a unified approach to the work on bounded convergence domains of several authors including Brudno, Copping, Mazur, Meyer–König, Orlicz, Petersen, Wilansky and Zeller. Petersen's theory of uniform summability is shown to be a statement concerning the compact subsets of a convergence

Received January 19, 1972. During the preparation of this manuscript the first author was partially supported by USAF Grant A569 1657.