ON THE ŠILOV AND BISHOP DECOMPOSITIONS OF A UNIFORM ALGEBRA

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In his article [1] E. Bishop succeeded in recovering a uniform algebra from its closed antisymmetric restriction algebras using the decomposition of the underlying space into its maximal sets of antisymmetry. Earlier progress in this direction had been made by G. E. Šilov [5] (see also [2; §44]) who introduced a coarser decomposition procedure which, iterated enough times, gives the Bishop decomposition. How many times is the subject of this paper. The outer limit turns out to be the first uncountable ordinal number σ^*(§1), and there is an example corresponding to each ordinal number up to this point (§2). In fact, with singly-generated algebras one can obtain precisely the finite and countable ordinal numbers, but the question of how many generators are required to reach σ^* is left open.

1. General results. Our undefined terms are standard and can be found, e.g., in the book of Stout [6] where one can also find a treatment of Bishop's theorem due to I. Glicksberg [3]. (A, X) shall denote a uniform algebra and $\mathcal{K} = \mathcal{K}(A)$ the collection of maximal sets of antisymmetry for A. In particular, A is assumed to separate the points of X, but it will be evident that this assumption is unnecessary.

There is an equivalence relation on X defined by $x \sim y$ if and only if $f(x) = f(y)$ for every real-valued function in A. The resulting family of equivalence classes will be denoted $\mathfrak{M} = \mathfrak{M}(A)$ and is the Šilov decomposition introduced in [5].

We shall use σ to denote ordinal numbers. We define inductively decompositions $\mathfrak{M}_\sigma = \mathfrak{M}_\sigma(A)$ of X into closed subsets as follows.

(1) $\mathfrak{M}_0 = \{X\}$.

(2) $\mathfrak{M}_{\sigma+1}$ consists of the sets of $\mathfrak{M}(A \mid E)$ for all $E \in \mathfrak{M}_\sigma$.

(3) If σ is a limit ordinal, $\mathfrak{M}_\sigma$ consists of the nonempty intersections $\bigcap_{\sigma'<\sigma} E_{\sigma'}$ where $E_{\sigma'} \in \mathfrak{M}_{\sigma'}$.

In order for step (2) to make sense one must verify by induction that $E \in \mathfrak{M}_\sigma$ is an intersection of peak sets for A and thus that $A \mid E$ is closed; the key step is the familiar lemma [6; 7.23] which asserts that an intersection of peak sets for the restriction of a uniform algebra to an intersection of its peak sets is again an intersection of peak sets for the uniform algebra. It is evident that $\mathfrak{M}_{\sigma'}$ is a refinement of $\mathfrak{M}_\sigma$ whenever $\sigma' > \sigma$ and that each member of $\mathfrak{M}_\sigma$ is a union of elements of $\mathcal{K}$ so that A may be recovered from its restrictions to elements of $\mathfrak{M}_\sigma$.

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