COMPACTNESS PROPERTIES OF TOPOLOGICAL GROUPS, II

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Introduction. The study in this paper is a continuation of that in [9]. Compactly generated subgroups of Cl $(P(G) \cap F(G))$ are proved to be relatively compact. Groups with the property $P(G) \subset F(G)$ have been studied. Results in [7] are generalized and sharpened. Some structure theorems of topological groups are presented.

Let G be a topological group. An element x of G is called *periodic* if x is contained in a compact subgroup of G. We denote by $P(G)$ the set of all periodic elements of G. An element x of G is called *conjugate-bounded* if the conjugacy class $C(x) = \{gxg^{-1}: g \in G\}$ is relatively compact in G. We write $F(G)$ for the set of all conjugate-bounded elements of G. Clearly $F(G)$ is a characteristic subgroup of G and in general $P(G)$ is not a subgroup of G. A topological group G is called an *FC-group* if $G = F(G)$. In [9] the author proved that for a locally compact group G, the set $P(G) \cap F(G)$ is a characteristic subgroup of G and a normal relatively compact subset Y of $P(G)$ generates a relatively compact normal subgroup $\langle Y \rangle$ of G. In this paper we shall extend this result. Here we prove that compactly generated subgroups of Cl $(P(G) \cap F(G))$ are relatively compact. In [7] V. I. Ušakov studied topological groups with the property that for every periodic element x in G, the quotient space G/N_G (Cl $\langle x \rangle$) of G over the normalizer N_G (Cl $\langle x \rangle$) of Cl $\langle x \rangle$ in G is compact. In this paper we replace the condition in [7] by the weaker condition $P(G) \subset F(G)$. We succeed in sharpening results in [7] and adding new ones. In general $P(G) \cap F(G)$ is not closed in G and an example is presented. We construct a locally compact group G such that $P(G) \cap F(G)$ is dense in G and not closed in G. Groups with dense $P(G)$ or dense $F(G)$ are studied.

1. Compactly generated subgroups of $\mathrm{Cl}(P(G) \cap F(G))$. In [9] it was proved that a relatively compact normal subset of $P(G)$ generates a relatively compact subgroup of G. In this section we shall extend the result further.

THEOREM 1.1. *Let G be a locally compact group and let L be the closure of $P(G) \cap F(G)$ in G. Then we have the following conditions.*

(i) *L has a compact open subgroup of L.*

(ii) *Every relatively compact subset Y of L generates a relatively compact subgroup $\langle Y \rangle$ of G.*

(iii) *L is contained in $P(G)$.*

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