ON EXTERIOR AND INTERIOR POINTS OF QUADRICS OVER A FINITE FIELD

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1. Let F denote a finite field of order q and of odd characteristic p, and let T_n be an n-dimensional projective space with base field F. Let P_n and P'_n denote two distinct $(n-1)$-dimensional hyperplanes of T_n defined by

$$b_0x_0 + b_1x_1 + \cdots + b_nx_n = 0$$

and

$$c_0x_0 + c_1x_1 + \cdots + c_nx_n = 0,$$

where at least one of the b_i and at least one of the c_i are nonzero respectively. If $n \geq 4$, a quadric of T_n defined by

$$a_0x_0^2 + a_1x_1^2 + \cdots + a_nx_n^2 = 0, \qquad a_0a_1 \cdots a_n \neq 0, \tag{1.1}$$

has at least one point in common with the $(n-2)$-dimensional hyperplane $P_n \cap P'_n$ [3; Theorem 3].

Let Q_n denote the quadric of T_n defined by (1.1). There is no loss in generality in assuming that Q_n is a diagonal form [2; §168]. If $\Psi(a)$ denotes the Legendre symbol in F, that is, $\Psi(a) = +1, -1$ or 0 according as a is a square, a nonsquare or zero in F, then the exterior of Q_n is defined to be the set of points $(x_0, x_1, \cdots, x_n)$ of T_n such that $\Psi(Q(x_0, x_1, \cdots, x_n)) = +1$ and the interior of Q_n is the set of points of T_n such that $\Psi(Q(x_0, x_1, \cdots, x_n)) = -1$ [1; §1].

For any set K of points of T_n let $N_E(K)$ denote the number of points of K in the exterior of Q_n and let $N_I(K)$ be the number of points of K in the interior of Q_n. L. Carlitz [1] determined $N_E(L_n)$ and $N_I(L_n)$, where L_n is an arbitrary $(n-1)$-dimensional hyperplane of T_n. In this paper we determine $N_E(P)$ and $N_I(P)$, where $P = P_n \cap P'_n$ (see Theorem 1). Moreover, as a direct consequence of Theorem 1 we find that either $N_E(P) = N_I(P)$ or $N_E(P) + N_I(P) = q^{n-2}$. Finally, we characterize ruled quadrics of T_3 (see Theorem 4).

2. Let $N'_E(P)$ denote the number of solutions $x_0, x_1, \cdots, x_n$ of the system of equations

$$\begin{aligned} b_0x_0 + b_1x_1 + \cdots + b_nx_n &= 0 \\ c_0x_0 + c_1x_1 + \cdots + c_nx_n &= 0 \end{aligned} \tag{2.1}$$

such that $\Psi(a_0x_0^2 + a_1x_1^2 + \cdots + a_nx_n^2) = 1$ and $N'_I(P)$ the number of solutions of (2.1) such that $\Psi(a_0x_0^2 + a_1x_1^2 + \cdots + a_nx_n^2) = -1$. Then

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