THE TENSOR PRODUCT OF COMPLETE BIORTHOGONAL SEQUENCES

By William H. Ruckle

1. Let X and Y be Banach spaces with conjugate spaces X^* and Y^* respectively. If (e_i, E_i) is a complete biorthogonal sequence in (X, X^*) and (e_i', E_i') a complete biorthogonal sequence in (Y, Y^*), then it is not hard to show that $(e_i \otimes e_j', E_i \otimes E_j')$ is a biorthogonal system in $(X \otimes Y, (X \otimes Y)')$, where $(X \otimes Y)'$ is the algebraic dual of $X \otimes Y$. In this paper some results are established with respect to the following problems.

1. When is $(e_i \otimes e_j', E_i \otimes E_j')$ a complete biorthogonal system in $(X \otimes_\theta Y, (X \otimes_\theta Y)^*)$?
2. When is $(e_i \otimes e_j', E_i \otimes E_j')$ a norming biorthogonal system in $(X \otimes_\theta Y, (X \otimes_\theta Y)^*)$, and what consequences follow when it is?
3. What is the nature of the multiplier algebra of $X \otimes_\theta Y$ with respect to $(e_i \otimes e_j', E_i \otimes E_j')$ when it is a complete biorthogonal system?
4. What enumerations of $(e_i \otimes e_j')$ are bases for $X \otimes_\theta Y$ assuming (e_i) is a basis for X and (e_i') is a basis for Y?

Here $X \otimes_\theta Y$ denotes the completion of $X \otimes Y$ with respect to a crossnorm θ, and $(X \otimes_\theta Y)^*$ denotes its conjugate space.

Problems 1 and 2 are treated for $\theta = \gamma$, the greatest crossnorm, in §4 and for $\theta = \lambda$, the least crossnorm, in §5. It is seen that Problem 1 for $\theta = \gamma$ is related to the question of whether X and Y have the approximation property (Theorem 4.2 and Proposition 4.3). Problem 2 for $\theta = \gamma$ is related in a somewhat lesser extent to the question of whether X and Y have the metric approximation property (Theorem 4.8 and Propositions 4.9, 4.10).

The only result given on Problem 3 is Proposition 6.1 which, on first glance, seems rather weak. But it is strong enough to yield Theorem 6.2, which is a result pertaining to Problem 4. Other work on Problem 4 was carried out in [4]. Another consequence of Proposition 6.1 is the result on the analog of Problem 4 with respect to strongly summable complete biorthogonal sequences. This is Theorem 6.3.

2. Notation and fundamental concepts. Throughout this paper (e_i, E_i) and (e_i', E_i') will denote complete biorthogonal sequences in (X, X^*) and (Y, Y^*) respectively, where X and Y are Banach spaces. The spaces X, X^*, Y, Y^* are respectively identified with the following spaces of sequences:

Received December 1, 1969.