# MAGNITUDE OF THE COEFFICIENTS OF THE CYCLOTOMIC POLYNOMIAL $F_{p a r}$, II 

By Marion Beiter

1. Introduction. Bang [1] in 1895 found that the coefficients of the monic cyclotomic polynomial $F_{p a r}$, where $p<q<r$ are odd primes, are not greater than $p-1$ in absolute value. No better upper bound has heretofore appeared. However, the author in [3] and Bloom in [4] simultaneously established $(p+1) / 2$ as the upper bound in the special case where $q$ and/or $r$ is congruent to $\pm 1$ modulo $p$. Bloom also proved that, for a coefficient of value $p-1$ to appear, a necessary condition is: $m q r+h q+r$ is not divisible by $p, h= \pm 1, m$ an integer such that $|m| \leq \min (3,(p-3) / 2)$. It is the purpose of this paper to show that $p-k$ when $p=4 k+1$ or $p-(k+1)$ when $p=4 k+3$ is a better general bound on these coefficients.
2. Preliminary notions. Let

$$
F_{p a r}(x)=\sum_{n=0}^{\varphi(\text { par })} c_{n} x^{n}
$$

Then, from [2],
(1)

$$
c_{n}=\sum(-1)^{\delta_{1}+\delta_{2}}
$$

where the summation (1) is over all partitions of $n$ such that $0 \leq n \leq \varphi(p q r)$, $\varphi(m)$ is the Euler $\varphi$-function and

$$
\begin{equation*}
n=a+\alpha p q \rightarrow \beta p r+\gamma q r+\delta_{1} q+\delta_{2} r \tag{2}
\end{equation*}
$$

$\delta_{1}, \delta_{2}=0,1 ; 0 \leq a<p ; 0 \leq \alpha<r ; 0 \leq \beta<q ; 0 \leq \gamma<p-1$.
Then $c_{n}=0$ when $n$ has no partition of the form (2). Otherwise the value of $c_{n}$ will be determined by the number of such partitions of $n$ and by the values of $\delta_{1}$ and $\delta_{2}$ in these partitions. Since the cyclotomic polynomial is symmetric we examine $c_{n}$ only for $n \leq[\varphi(p q r)] / 2$. With this restriction on $n, \gamma$ in (2) is not greater than $(p-3) / 2$. The possible values of the $\delta$ 's suggest the following notation for partitions of $n$ in the form (2) [3]:

$$
\left.\begin{array}{ll}
\delta_{1}=\delta_{2}=0, & P_{1 i}=a_{1 i}+\alpha_{1 i} p q+\beta_{1 i} p r+i q r \\
\delta_{1}=\delta_{2}=1, & P_{2 i}=a_{2 i}+\alpha_{2 i} p q+\beta_{2 i} p r+i q r+q+r \\
\delta_{1}=1, \delta_{2}=0, & P_{3 i}=a_{3 i}+\alpha_{3 i} p q+\beta_{3 i} p r+i q r+q \\
\delta_{1}=0, \delta_{2}=1, & P_{4 i}=a_{4 i}+\alpha_{4 i} p q+\beta_{4 i} p r+i q r+r
\end{array}\right\}, \quad i=0,1, \cdots,(p-3) / 2 .
$$

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